Ministry of Higher Education
Higher Institute For Engineering
And Modern Technology Marg

Mechanical design

MED211

Assicate Prof.Dr. Sayed Elnggar
Dr.Gomaa A.Abdelrazek

2026

Contents

No	Title	Page
	CHAPTRR.1:	
1.1	Basic principles in design, ASME boilers and	4
1.2	pressure vessels ,piping codes,	
1.3	Piping, Drums, and Headers design	10
1.4	Material selection	23
	CHAPTRR.2	
2.1	Types of loads	23
2.2	Failure theories	25
2.3	Design of thin cylinders and application of finite Element Method	43
	Chapter.3	105
	Theory of tribology	75
3.1	Wear	76
3.2	Design and selection of bearings and seals ,	83



Mechanical design 211

3 Cr. = [2Let + 2Tut +1Lab] hrs.

Prerequisite: MED 113

Basic principles in design, ASME boilers and pressure vessels ,piping codes, Material selection, Types of loads failure theories, Design of thin cylinders,Application of finite Element Method, theory of tribology, Design and selection of bearings and seals , Course project.

مكص ٢١١ تصميم ميكانيكى

المبادى الاساسية للتصميم، تصميم الغلايات واوعية الضغط، اكواد المواسير، اختيار المواد، انواع الاحمال، نظرياتالانهيار، تصميم المواسير والاوعية رفعية الجدار، تطبيق طريقة العناصر المحدودة فى حسابات التصميم،نظريات الترايبولوجى، تصميم واختيار كراسى المحاور وموانع التسرب، مشروع مقرر.

Laboratory:

Practice on computer graphics packages such as AUTOCAD,SOLIDWORKS,…. Etc with case studies.

practice on finite Element Method(FEM) computer package such as COSMOS,APAQUES, NASTRAN,…. Etc with practical examples

Practice on Bearings and seals selection from standard tables

Computer oriented Design Project.

References:

Joseph shigley, Mischke, Mechanical Engineering Design, , Tata McGraw -Hill,6thEd., 2004

shigley,Richard Budynaskeith Nisbett, Mechanical Engineering Design(Mcgraw -Hill Series in Mechanical Engineering),McGraw - Hill science / Engineering /Math, 9thEd ., 2010

Robert L.Mott, Machine Elements in Mechanical Design , prentice Hall, 4thEd ., 2003

Pahl and W. Beitz, Engineering Design: Asystematic approach, Springer - Verlag London, 3rdEd., 2007

CHAPTRR.1:

1-Basic principles of machine design

Introduction

Design is essentially a decision-making process. If we have a problem, we need to design a solution. In other words, to design is to formulate a plan to satisfy a particular need and to create something with a physical reality. Consider for an example, design of a chair. A number of factors need be considered first:

(a) The purpose for which the chair is to be designed such as whether it is to be used as an easy chair, an office chair or to accompany a dining table.

(b) Whether the chair is to be designed for a grown up person or a child.

(c) Material for the chair, its strength and cost need to be determined.

(d) Finally, the aesthetics of the designed chair.

Factors to be considered in machine design

Some of these factors are as follows:

(a) What device or mechanism to be used.

(b) Material

(c) Forces on the elements

(d) Size, shape and space requirements. The final weight of the product is also a major concern.

(e) The method of manufacturing the components and their assembly.

(f) How will it operate?

(g) Reliability and safety aspects

(h) Maintenance, cost and aesthetics of the designed product.

2-The ASME Boiler and Pressure Vessel

The ASME - American Society of Mechanical Engineers - International Boiler and Pressure Vessel

Code is made of 12 sections and contains over 15 divisions and subsections.

1.1 - Code Sections:

I. Power Boilers;

II. Materials;

III. Rules for Construction of Nuclear Facility Components;

IV. Heating Boilers;

V. Nondestructive Examination;

VI. Recommended Rules for the Care and Operation of Heating Boilers;

VII. Recommended Guidelines for the Care of Power Boilers;

VIII. Pressure Vessels;

IX. Welding and Brazing Qualifications;

X. Fiber-Reinforced Plastic Pressure Vessels;

XI. Rules for In-service Inspection of Nuclear Power Plant Components;

XII. Rules for Construction and Continued Service of Transport Tanks.

• Boilers constructed per ASME Sections I (HP) & IV (LP & HW)
• The boiler package consists of pressure vessel, burner &controls (BMS & CCS)

Section I

- High pressure - Steam boilers above 15 psi. Hot water boilers above 160 psi (hydrostatic pressure) and/or 250^0 F outlet temperature



Section IV

- Low pressure - Steam boilers less than 15 psi. Hot water boilers less than 160 psi and/or 250^0 F. outlet temperature



The Boiler Package

- Pressure vessel
- Burner
- Controls

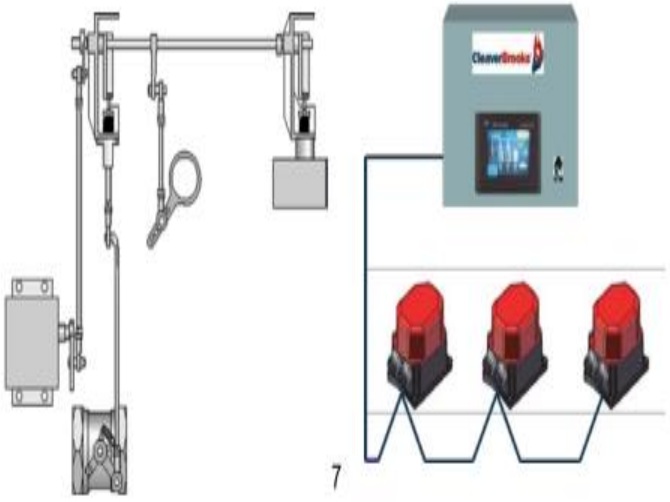
Pressure Vessel



Burner



Combustion Controls



Burner Management

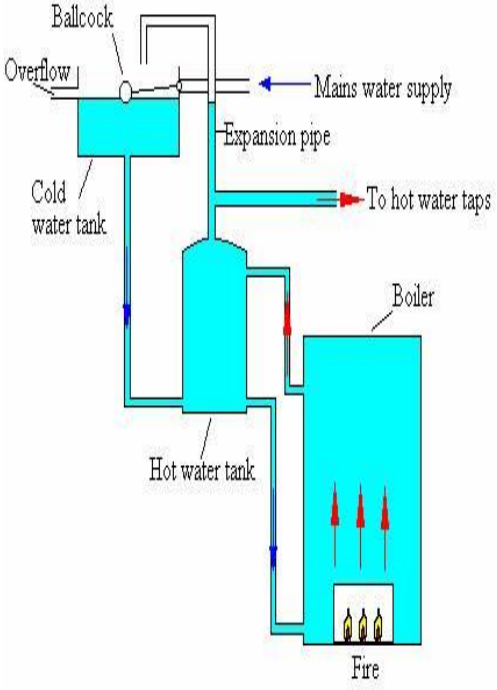


boiler definition

- A water containing vessel in which water is heated under pressure
- Transfer heat from a fuel source (oil, gas, coal) into steam
- The steam or hot fluid circulates out of the boiler for use in various process or heating appliances. Boilers are classified into fire tube or water tube boilers depending on whether the heat source is inside or outside the tubes.
- The goal in both cases is to maximize the heat transfer between the water and the hot gases heating it.

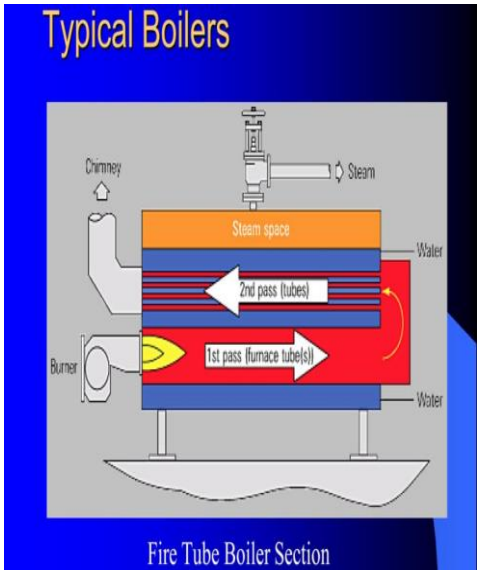
Boiler work

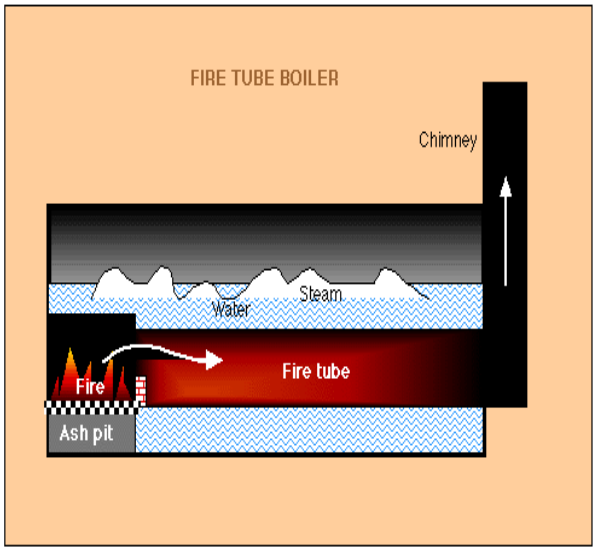
- Closed system: when 100% of the steam produced is returned to be reused.
- Examples of closed systems are close steam heating, hot water heating, and “one-pipe” systems.
- Open system: a system that does not return the condensate.



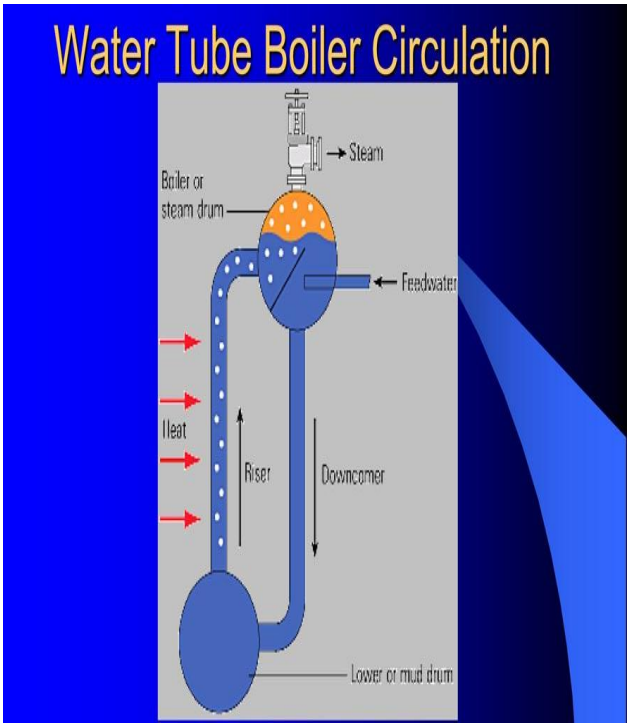
Fire Tube Boilers

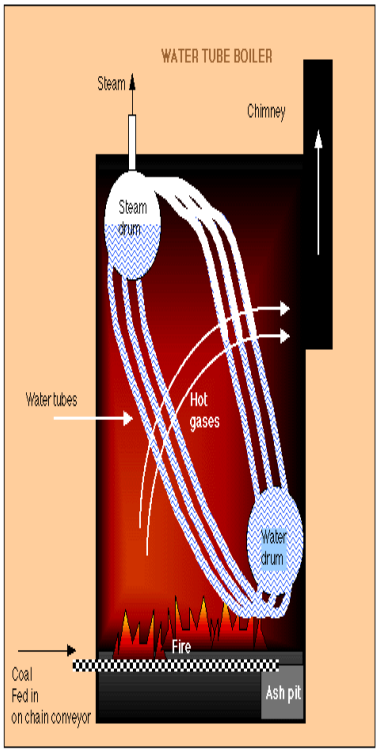
- The combustion gases pass inside boiler tubes, and heat is transferred to water on the shell side.





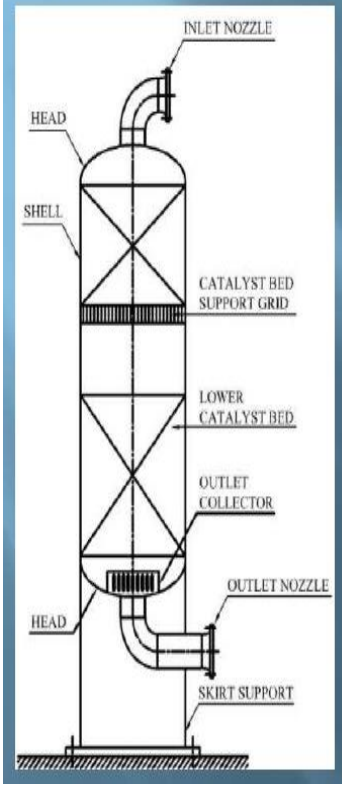
- **Water Tube Boilers**
- Boiler water passes through the tubes while the exhaust gases remain in the shell side, passing over the tube surfaces.



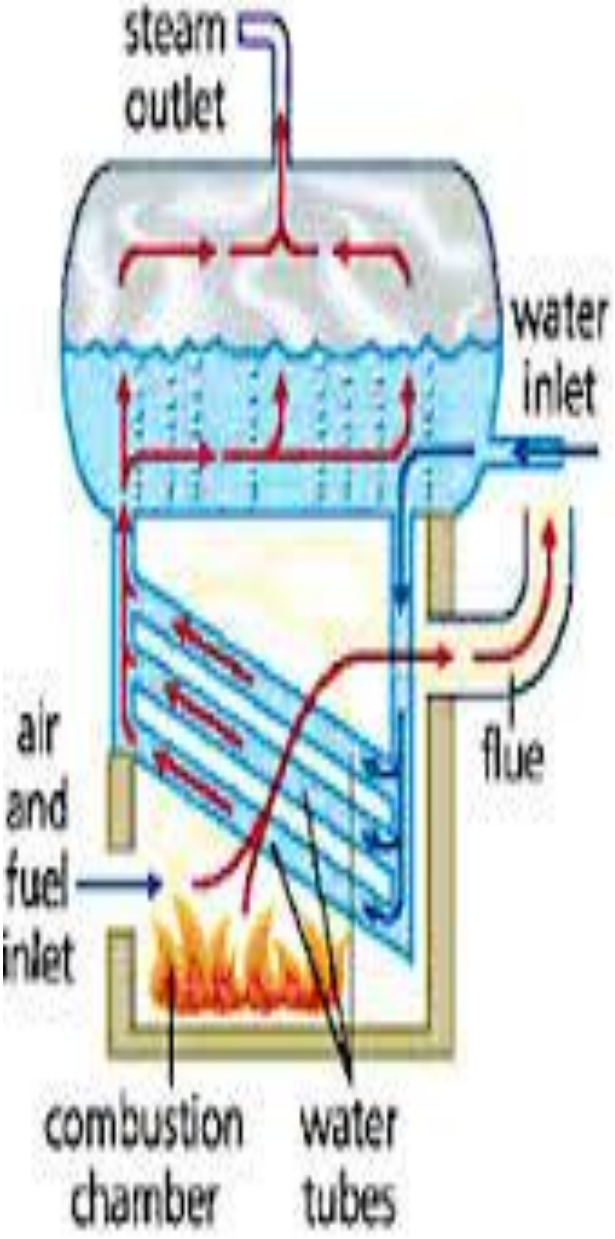


3-Piping, Drums, and Headers design

TYPICAL COMPONENTS



1. Cylindrical or Spherical Shell
2. Formed Heads
3. Blind Flanges, Cover Plates, Flanges
4. Openings And Nozzles
5. Supports



ASME SECTION I - POWER BOILERS: Types, Design, Fabrication, Inspection & Repair:

Provides requirements for construction of power, electric, and miniature boilers; high temperature water
boilers used in stationary service; and power boilers used in locomotive, portable, and traction service . Rules allow the use of the V, A, M, PP, S and E symbol stamps.

These rules are applicable to boilers in which the steam is generated at pressures exceeding 15 psig, and high temperature water boilers for operation at pressures exceeding 160 psig and/or temperatures exceeding 250 °F. This code covers Power Boiler Superheaters, Economizers, and other pressure parts connected directly to the boiler. Valves are considered as part of the scope in Section 1.

ASME SECTION I – Boiler Tubes up to and including 5 inches O.D. (125 mm):

a) The minim ...SME, use equation bel...

$$t = \frac{PD}{2S + P} + 0.005D + e \qquad 1.1$$

b) To calculate the **Maximum Allowable Working Pressure (MAWP)**:

$$P = S\left[\frac{2t - 0.01D - 2e}{D - (t - 0.005D - e)}\right] \qquad 1.2$$

Where:

t = Minimum Design Wall Thickness (in);
P = Design Pressure (psi);
D = Tube Outside Diameter (in);
e = Thickness Factor (0.04 for expanded tubes; 0 = for strength welded tubes);
S = Maximum Allowable Stress According to ASME Section II, Table 1A.

ASME SECTION I & SECTION VIII – Fundamentals:

The formulae in ASME Section I and Section VIII are used to determine the minimum required thickness and design pressure of piping, tubes, drums and headers using the Maximum Allowable Working Pressure (MAWP). However,that these formulae may be also used for
calculating wall thickness of tubes and pipes under internal pressure.

Design:

The ASME Boiler Code Section I, as well as Section VIII, requires longitudinal and circumferential butt joints to be examined by full radiograph.

When the vessel design is required fully radiographed longitudinal butt-welded joint,the cylindrical shell will have a joint efficiency factor (E = 1.0). This factor corresponds to a safety factor (or material quality factor) of 3.5 in the parent metal.

When the vessel design is required non-radiographed longitudinal butt-welded joints the vessel
will have a joint efficiency factor (E = 0.7, which corresponds to a safety factor of 0.5 resulting
in an increase of 43% in the thickness of the plates.

Pressure Vessels Maximum Allowable Stress Values:

The maximum allowable stress values to be used in the calculation of the vessel's wall thickness
are given in the ASME Code for many different materials. These stress values are a function of temperature.

Piping Codes

The ASME-ANSI B31 Codes:

ASME B31 was earlier known as ANSI B31.

The B31 Code for Pressure Piping, covers Power Piping , Fuel Gas Piping, Process Piping, Pipeline Transportation Systems for Liquid Hydrocarbons and Other Liquids, Refrigeration Piping and Heat Transfer Components and Building Services Piping.

Piping consists of pipe, flanges, bolting, gaskets, valves, relief devices, fittings and the pressure containing parts other piping components.

It also includes hangers and supports, and other equipment items necessary to prevent overstressing the pressure containing parts.

It does not include support structures such as frames of buildings, buildings stanchions or foundations.

ASME SECTION I – Piping, Drums, and Headers:

The following formulae are found in ASME Section I. The information for piping,drums, or headers may be given with either the inside I or outside D measurements.

a) Using the **outside diameter**:

$$t = \frac{PD}{2SE + 2yP} + C \qquad 2.1$$

$$P = \frac{2SE(t - C)}{D - (2y)(t - C)} \qquad 2.2$$

b) Using the **inside radius**:

$$t = \frac{PR}{SE - (1 - y)P} + C \qquad 2.3$$

$$P = \frac{SE(t - C)}{R + (1 - y)(t - C)} \qquad 2.4$$

t = Minimum Design Wall Thickness (in);
P = Design Pressure (psi);
D = Tube Outside Diameter (in);
R = Tube Radius (in);
E = Tube Welding Factor (1.0 for seamless pipe; 0.85 = for welded pipe);
y = Wall Thickness Welding Factor (0.4 for 900°F & lower; 0.7 for 950°F & up);
C = Corrosion Allowance (0 for no corrosion; 0.0625 in. commonly used; 0.125 in. maximum);
S = Maximum Allowable Stress According to ASME Section II, Table 1A.

ASME SECTION VIII – Division 1, Division 2, Division 3:

The ASME Section VIII, rules for fired or unfired pressure vessels, is divided into three divisions applicable to design, fabrication, inspection, testing, and certification. The formulae and allowable stresses presented in this sketch are only for Division 1, the main code. It contains mandatory and nonmandatory appendices detailing supplementary design criteria, nondestructive examination and inspection acceptance standards. The use of the U, UM and UV Code symbol stamps are also included.

SECTION VIII - Thin Cylindrical Shells:

The formulae in ASME Section VIII, Division 1, used for calculating the wall thickness and design pressure of pressure vessels, are:

a) Circumferential Stress (longitudinal welds):

When, P < 0.3[illegible]:

$$t = \frac{PR}{(SE - 0.6P)} \qquad 1.3$$

Or

$$P = \frac{SEt}{(R + 0.6t)} \qquad 1.4$$

(R = Internal Radius)

a) Longitudinal Stress (circumferential welds):

- When, P < 1.25SE:

$$t = \frac{PR}{(2SE + 0.4P)} \qquad 1.5$$

Or

$$P = \frac{2SEt}{(R - 0.4t)} \qquad 1.6$$

(R = Internal Radius)

SECTION VIII - Thick Cylindrical Shells:

For internal pressures higher than 3,000 psiunder special considerations .

As the ratio of t/R increases beyond 0.5, an accurate equation is required to determine the thickness.

The formulae in ASME Appendix 1, Supplementary Design Formulas used for calculating thick wall and design pressure, are:

a) For longitudinal welds:

When, $P > 0.385SE$:

$$t = R\left(Z^{\frac{1}{2}} - 1\right) \quad \text{Where} \quad Z = \frac{(SE + P)}{(SE - P)} \qquad 1.7$$

And:

$$P = SE\left[\frac{(Z - 1)}{(Z + 1)}\right] \quad \text{Where} \quad Z = \left[\frac{(R + t)}{R}\right]^2 \qquad 1.8$$

b) For circumferential welds:

When, $P > 1.25SE$:

$$t = R\left(Z^{\frac{1}{2}} - 1\right) \quad \text{Where} \quad Z = \left(\frac{P}{SE}\right) + 1 \qquad 1.9$$

And:

$$P = SE(Z - 1) \quad \text{Where} \quad Z = \left[\frac{(R + t)}{R}\right]^2 \qquad 1.10$$

Where:

R = Design Radius (in.):

Z = Dimensionless Factor.

ASME I and ASME-ANSI B31 – Pressure Piping - Minimum Wall Thickness:

According to ASME Section I and ANSI B31, the minimum thickness of

$$t = \frac{PD}{2SE + 2yP} + C \qquad 1.12$$

t (min)= Minimum wall thickness required (in);
P = Design pressure (psig);
D = Outside diameter of pipe (in);
S = Allowable stress in pipe material (psi);
E = Longitudinal joint factor - E = 1.0 for seamless pipe, E = 0.85 for ERW pipe;
C = Corrosion allowance, typically 0.05 in.

y = Wall thickness coefficient in ASME according to Table 304.1.1 for ferritic steels, where:

y = **0.4** for T <= 900 °F;
y = **0.5** for 900 < T <= 950 °F;
y = **0.7** for 950 < T <= 1000 °F.

ASME SECTION I: Dished Heads Formulae:

Dished heads can be manufactured using a combination of processes, spinning & flanging, where the spherical radius is made via the spinning process and the knuckle is created under the flanging method.

Flanged and dished heads can be formed in a size range from 4 in to 300 in diameter and in thickness range of 14 Gauge to 1-1/2” thick. Pressure vessel heads and dished ends are essentially the same the end caps of a pressure vessel tank or an industrial boiler, supplied with a flanged edge to make it easier for the fabricator to weld the head to the main body of the tank.

Blank, Unstayed Dished Heads:

According to ASME ,Paragraph PG-29.1 states that the thickness of a blank, unstayed dished head with the pressure on the concave side, when it is a segment of a sphere, shall be calculated by the following formula:

$$t = \frac{5PL}{4.8S} \qquad 1.11$$

Where:

t = Minimum thickness of head (in);

P = maximum allowable working pressure (psi);

L = Concave side radius (in);

S = Maximum Allowable Working Stress (psi).

Paragraph PG-29.2 states: "The **radius** to which the head is dished shall be **not greater than the out-side diameter of the flanged portion of the head**. Where two radii are used, the longer shall be taken as the value of ***L*** in the formula."

Seamless or Full-Hemispherical Head:

The thickness of a blank, unstayed, full-hemispherical head with the pressure on the concave side shall be calculated by the following formula:

$$t = \frac{PL}{2S - 0.2P} \qquad 1.13$$

t = Minimum thickness of head (in)
P = Maximum Allowable Working Pressure (psi)
L = Radius to which the head was formed (in)
S = Maximum Allowable Working Stress (psi)

Note: The above formula **shall not be used** when the required thickness of the head given by the formula **exceeds 35.6% of the inside radius**. Instead, use the following formula:

$$t = L\left(Y^{\frac{1}{3}} - 1\right) \quad \text{where} \quad Y = \frac{2(S + P)}{2S - P} \qquad 1.14$$

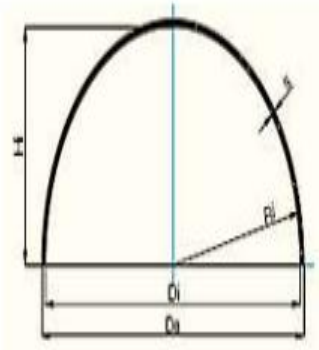
ASME SECTION VIII – DIVISION 1: Dished Head Formulae:

Flanged and dished heads can be formed from carbon steel, stainless steel, and other ferrous and non-ferrous metals and alloys. Flanged and dished heads can be formed in a size range from 4 inches to 300inches diameter and in thickness range of 1/8 in. to 1 ½ inches thick.

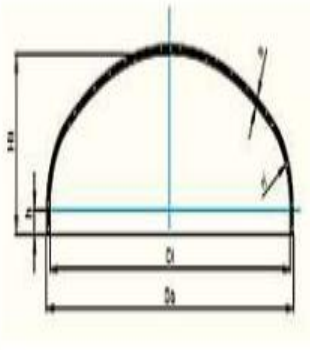
The ASME Section VIII – Division1 determines the rules for dished heads. The most common configurations are spherical, hemispherical, elliptical (or ellipsoidal) and torispherical shapes.

Dished Head Shapes:

How the shapes are, make some confusion for beginners and even professionals users of ASMESection VIII. To cast a little light on these subjects see the resume below:

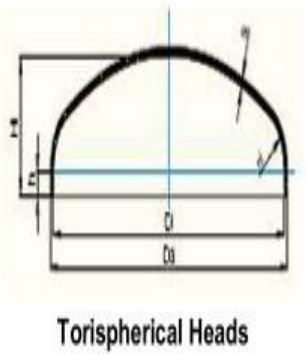


Spherical or Hemispherical Heads



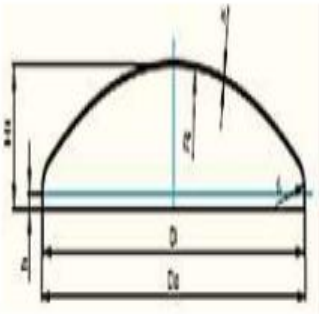
Semi-Elliptical Heads (2:1)

Elliptical or Ellipsoidal Heads (1.9:1)

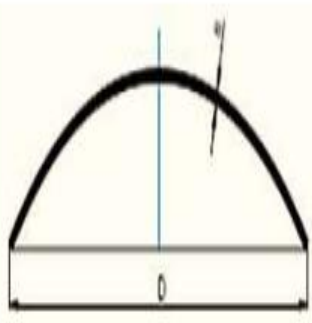


Torispherical Heads

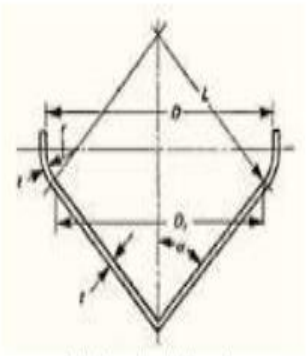
Flanged and Dished Heads



Flat Dished Heads & Non-Standard 80-10 Dished Heads



Dished Discs



Toriconical Heads

Spherical or Hemispherical Heads

a) When $t < 0.356R$ or $P < 0.665SE$ - *(Thin Spherical or Hemispherical Heads):*

$$t = \frac{PR}{2SE - 0.2P} \qquad 2.1$$

And,

$$P = \frac{2SEt}{R + 0.2t} \qquad 2.2$$

b) When t > 0.356R or P > 0.665SE – (Thick Spherical or Hemispherical Heads):

$$t = R\left(Y^{\frac{1}{3}} - 1\right) \quad where \quad Y = \frac{2(SE + P)}{2SE - P} \qquad 2.3$$

And,

$$P = 2SE\left(\frac{Y - 1}{Y + 2}\right) \quad \text{where} \quad Y = \left(\frac{R + t}{R}\right)^3 \qquad 2.4$$

Materials Selection for Mechanical Design

Need Method for Early Material Selection: Ashby Methodology*

Four basic steps

1. **Translation:** ***express design requirements as constraints & objectives***
2. **Screening:** ***eliminate materials that cannot do the job***
3. **Ranking:** ***find the materials that do the job best***
4. **Supporting information:** ***explore pedigrees of top-ranked candidates***

First Step: Translation

"Express design requirements as constraints and objectives"

Using design requirements, analyze four items:

Q Function: What does the component do?

- *Do not limit options by specifying implementation w/in function*

Q Objective: What essential conditions must be met?

- *In what manner should implementation excel?*

Q Constraints: What is to be maximized or minimized?

- *Differentiate between binding and soft constraints*

Q Free variables: Which design variables are free?

- *Which can be modified?*
- *Which are desirable?*

CHAPTER.2.
Types of loads and Failure theories

Introduction

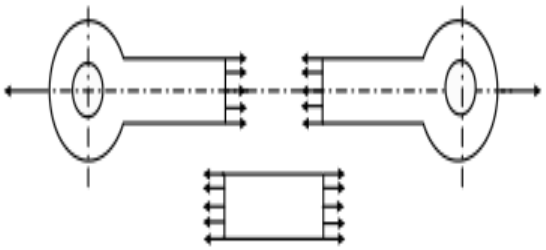
Machine parts fail when the stresses induced by external forces exceed their strength. The external loads cause internal stresses in the elements and the component size depends on the stresses developed. Stresses developed in a link subjected to uniaxial loading is shown in figure-.1. Loading may be due to:

a) The energy transmitted by a machine element.
b) Dead weight.
c) Inertial forces.
d) Thermal loading.
e) Frictional forces.

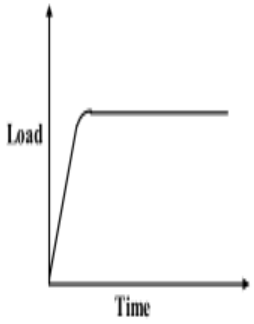
In another way, load may be classified as:

a) Static load- Load does not change in magnitude and direction and normally increases gradually to a steady value.
b) Dynamic load- Load may change in magnitude for example, traffic of varying weight passing a bridge.Load may change in direction, for example, load on piston rod of a double acting cylinder.

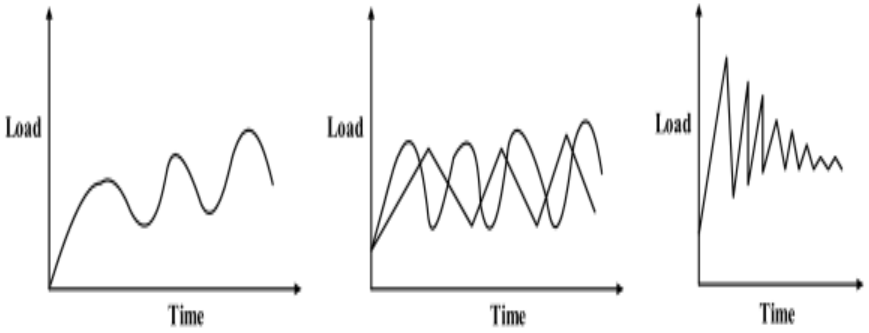
Vibration and shock are types of dynamic loading. Figure-.2 shows load v_s time characteristics for both static and dynamic loading of machine elements.



1. - Stresses developed in a link subjected to uniaxial loading



Static Loading



Dynamic Loading

.2 - Types of loading on machine elements.

An allowable stress is set considerably lower than the ultimate strength. The ratio of ultimate to allowable load or stress is known as factor of safety i.e.

$$\frac{\text{Ultimate Stress}}{\text{Allowable Stress}} = \text{F.S.}$$

The ratio must always be greater than unity. It is easier to refer to the ratio of stresses since this applies to material properties.

2-Theories of failure

When a machine element is subjected to a system of complex stress system, it is important to predict the mode of failure so that the design methodology may be based on a particular failure criterion. Theories of failure are essentially a set of failure criteria developed for the ease of design.

In machine design an element is said to have failed if it ceases to perform its function. There are basically two types of mechanical failure:

(a) Yielding - This is due to excessive inelastic deformation rendering the machine part unsuitable to perform its function. This mostly occurs in ductile materials.

(b) Fracture - in this case the component tears apart in two or more parts. This mostly occurs in brittle materials.

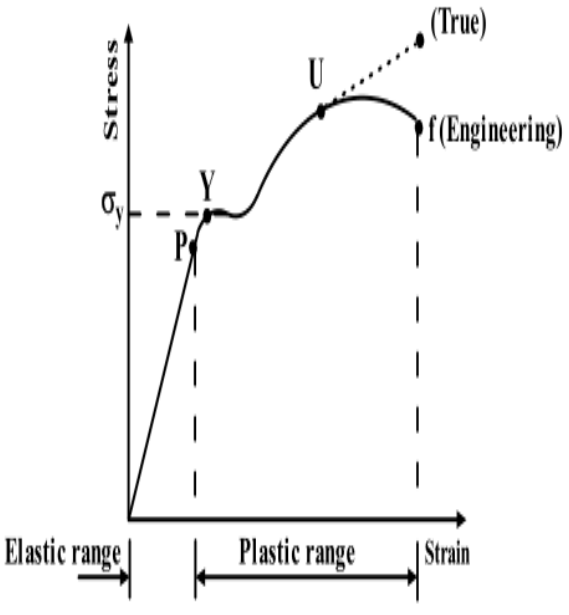
There is no sharp line of demarcation between ductile and brittle materials.

However a rough guideline is that if percentage elongation is less than 5% then the material may be treated as brittle and if it is more than 15% then the material is ductile.

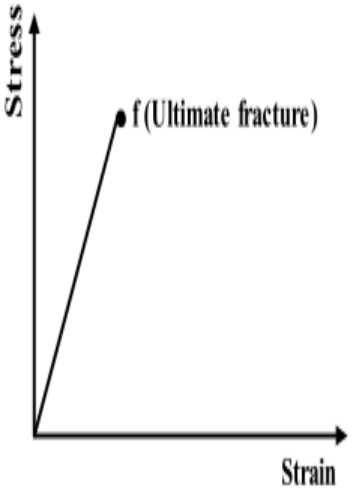
However, there are many instances when a ductile material may fail by fracture. This may occur if a material is subjected to

(a) Cyclic loading.
(b) Long term static loading at elevated temperature.
(c) Impact loading.
(d) Work hardening.
(e) Severe quenching.

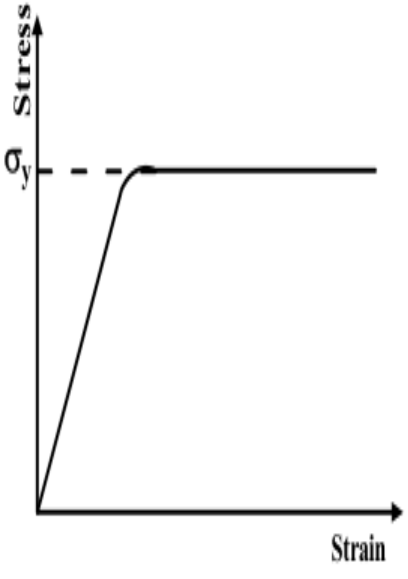
Yielding and fracture can be visualized in a typical tensile test as shown in the clipping- Typical engineering stress-strain relationship from simple tension tests for same engineering materials are shown in figure- 3.



3. (a) Stress-strain diagram for a ductile material



(b) Stress-strain diagram for a brittle material.



(c) Stress-strain diagram for an elastic – perfectly plastic material.

Yield criteria

There are numerous yield criteria, going as far back as Coulomb (1773). Many of these were originally developed for brittle materials but were later applied to ductile materials. Some of the more common ones will be discussed briefly here.

1 Maximum principal stress theory (Rankine theory)

-According to this, if one of the principal stresses σ 1 (maximum principal stress), σ 2 (minimum principal stress) or σ 3 exceeds the yield stress, yielding would occur.

-In a two dimensional loading situation for a ductile material where tensile and compressive yield stress are nearly of same magnitude

$$\sigma 1 = \pm \sigma y$$

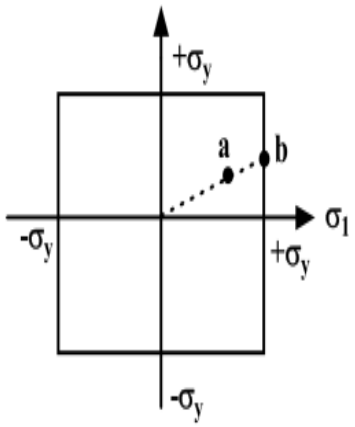
$$\sigma 2 = \pm \sigma y$$

-Using this, a yield surface may be drawn, as shown in figure-.4.1 .Yielding occurs when the state of stress is at the boundary of the rectangle.

-Consider, for example, the state of stress of a thin walled pressure vessel. Here σ 1 = 2σ 2 , σ 1 being the circumferential or hoop stress and σ 2 the axial stress.

-As the pressure in the vessel increases the stress follows the dotted line. At a point (say) a, the stresses are still within the elastic limit but at b, σ 1 reaches σ y although σ 2 is still less than σ y . Yielding will then begin at point b.

This theory of yielding has very poor agreement with experiment. However,the theory has been used successfully for brittle materials.



.4.1. Yield surface corresponding to maximum principal stress theory

2 Maximum principal strain theory (St. Venant's theory)

According to this theory, yielding will occur when the maximum principal strain just exceeds the strain at the tensile yield point in either simple tension or compression. If ε 1 and ε 2 are maximum and minimum principal strains corresponding to σ 1 and σ 2, in the limiting case

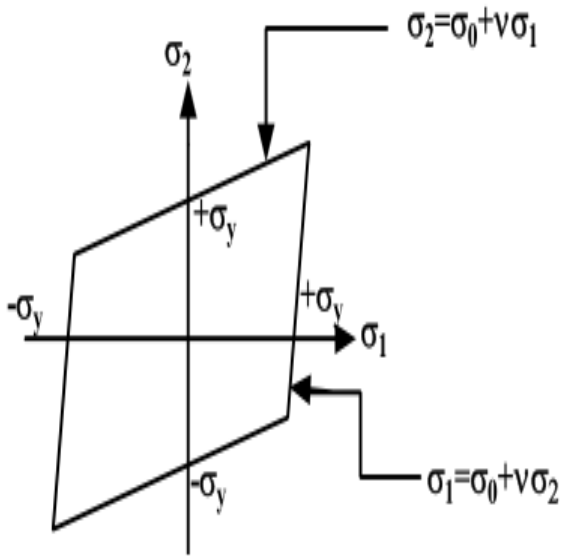
$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \qquad |\sigma_1| \geq |\sigma_2|$$

$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \qquad |\sigma_2| \geq |\sigma_1|$$

$$\text{This gives, } E\varepsilon_1 = \sigma_1 - \nu\sigma_2 = \pm\sigma_0$$

$$E\varepsilon_2 = \sigma_2 - \nu\sigma_1 = \pm\sigma_0$$

The boundary of a yield surface in this case is thus given as shown in **figure. 4.2.**



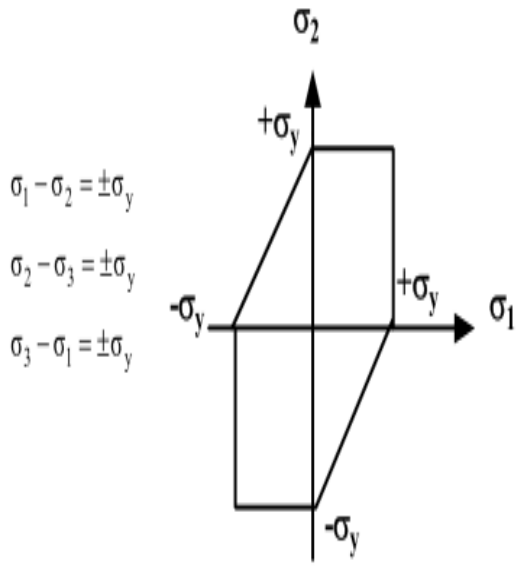
4.2. *Yield surface corresponding to maximum principal strain theory*

3 Maximum shear stress theory (Tresca theory)

According to this theory, yielding would occur when the maximum shear stress just exceeds the shear stress at the tensile yield point. At the tensile yield point σ 2 = σ 3 = 0 and thus maximum shear stress is σ y /2. This gives us six conditions for a three-dimensional stress situation:

(a) Maximum shear stress theory,

$$\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$



4.3. *Yield surface corresponding to maximum shear stress theory*

In a biaxial stress situation (figure-4.3) case, σ 3 = 0 and this:

gives

$$\sigma_1 - \sigma_2 = \sigma_y \quad \text{if } \sigma_1 > 0, \sigma_2 < 0$$

$$\sigma_1 - \sigma_2 = -\sigma_y \quad \text{if } \sigma_1 < 0, \sigma_2 > 0$$

$$\sigma_2 = \sigma_y \quad \text{if } \sigma_2 > \sigma_1 > 0$$

$$\sigma_1 = -\sigma_y \quad \text{if } \sigma_1 < \sigma_2 < 0$$

$$\sigma_1 = -\sigma_y \quad \text{if } \sigma_1 > \sigma_2 > 0$$

$$\sigma_2 = -\sigma_y \quad \text{if } \sigma_2 < \sigma_1 < 0$$

This criterion agrees well with experiment.

In the case of pure shear, $\sigma_1 = -\sigma_2 = k$ (say), $\sigma_3 = 0$

and this gives $\sigma_1 - \sigma_2 = 2k = \sigma_y$

This indicates that yield stress in pure shear is half the tensile yield stress

4 Maximum strain energy theory (Beltrami's theory)

According to this theory failure would occur when the total strain energy absorbed at a point per unit volume exceeds the strain energy absorbed per unit volume at the tensile yield point. This may be given

$$\frac{1}{2}\left(\sigma_1\varepsilon_1+\sigma_2\varepsilon_2+\sigma_3\varepsilon_3\right)=\frac{1}{2}\sigma_y\varepsilon_y$$

$$\frac{1}{2}\left(\sigma_1\varepsilon_1+\sigma_2\varepsilon_2+\sigma_3\varepsilon_3\right)=\frac{1}{2}\sigma_y\varepsilon_y$$

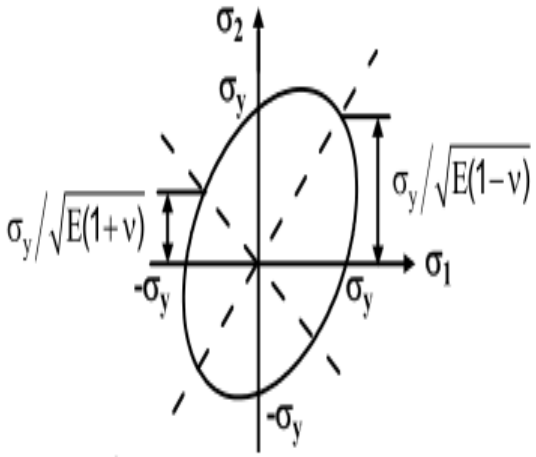
Substituting, ε_1, ε_2, ε_3 and ε_y in terms of stresses we have

$$\sigma_1^2+\sigma_2^2+\sigma_3^2-2\upsilon\left(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1\right)=\sigma_y^2$$

This may be written as

$$\left(\frac{\sigma_1}{\sigma_y}\right)^2+\left(\frac{\sigma_2}{\sigma_y}\right)^2-2\nu\left(\frac{\sigma_1\sigma_2}{\sigma_y^2}\right)=1$$

This is the equation of an ellipse and the yield surface is shown in **figure-4.4.**



***4.4.** Yield surface corresponding to Maximum strain energy theory.*

It has been shown earlier that only distortion energy can cause yielding but in the above expression at sufficiently high hydrostatic pressure $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$ (say), yielding may also occur.

From the above we may write $\sigma^2(3-2\nu)=\sigma_y^2$ and if $\nu \sim 0.3$, at stress level lower than yield stress, yielding would occur. This is in contrast to the experimental as well as analytical conclusion and the theory is not appropriate.

5 Distortion energy theory(von Mises yield criterion)

According to this theory yielding would occur when total distortion energy absorbed per unit volume due to applied loads exceeds the distortion energy absorbed per unit volume at the tensile yield point. Total strain energy E T and strain energy for volume change E V can be given as

$$E_T = \frac{1}{2}\left(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3\right) \text{ and } E_V = \frac{3}{2}\sigma_{av}\varepsilon_{av}$$

Substituting strains in terms of stresses the distortion energy can be given as

$$E_d = E_T - E_V = \frac{2(1+\nu)}{6E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1\right)$$

At the tensile yield point, $\sigma_1 = \sigma_y$, $\sigma_2 = \sigma_3 = 0$ which gives

$$E_{dy} = \frac{2(1+\nu)}{6E}\sigma_y^2$$

The failure criterion is thus obtained by equating E_d and E_{dy}, which gives

$$\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2 = 2\sigma_y^2$$

In a 2-D situation if $\sigma_3 = 0$, the criterion reduces to

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_y^2$$

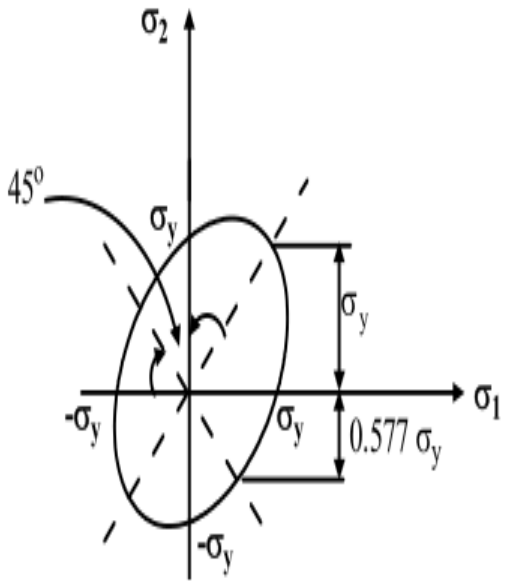
$$\text{i.e. } \left(\frac{\sigma_1}{\sigma_y}\right)^2 + \left(\frac{\sigma_2}{\sigma_y}\right)^2 - \left(\frac{\sigma_1}{\sigma_y}\right)\left(\frac{\sigma_2}{\sigma_y}\right) = 1$$

This is an equation of ellipse and the yield surface is shown in **figure .4.5.**

Maximum distortion energy theory

$$\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_1 - \sigma_3\right)^2 = 2\left(\sigma_Y / F.S\right)^2$$

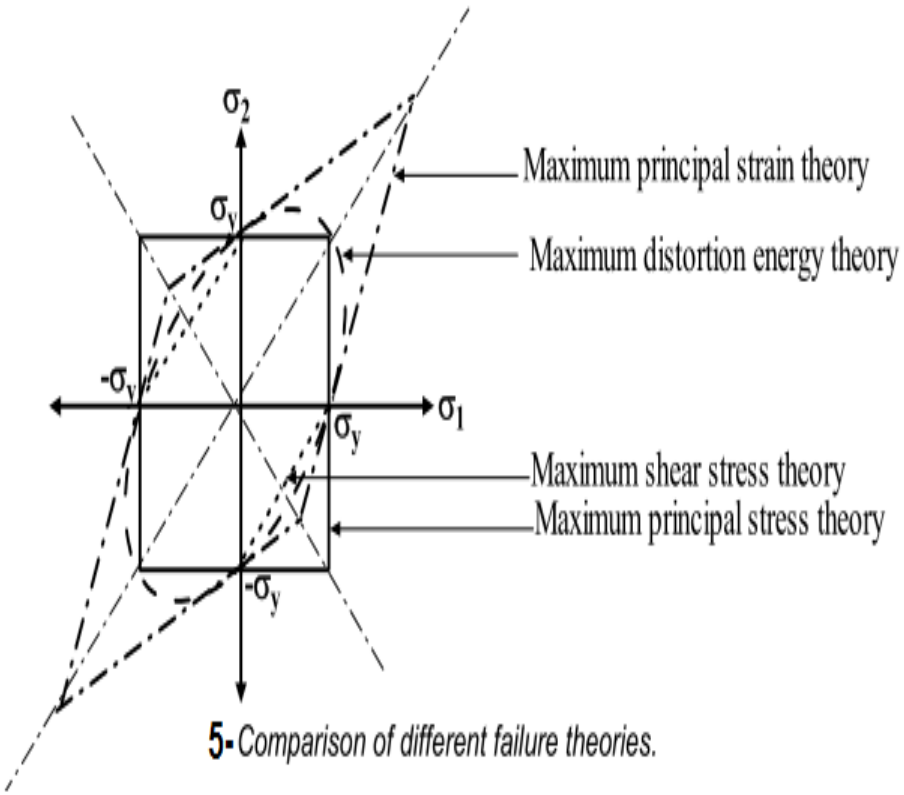
This theory agrees very well with experimental results and is widely used for ductile materials.



.4.5. Yield surface corresponding to von Mises yield criterion.

.5 Superposition of yield surface

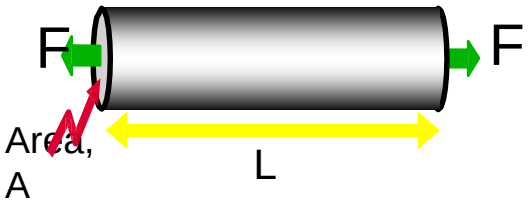
A comparison among the different failure theories can be made by superposing the yield surfaces as shown in **figure- 3.1.5.1**.



5- *Comparison of different failure theories.*

Identifying Desirable Characteristics
Example: Materials for a Light, Strong Tie

- **Function:**
  - **Support a tension load**
- **Objective:**
  - **Minimize mass**
- **Constraints:**
  - **Length specified**
  - **Carry load F, w/o failure**
- **Free variables:**
  - **Cross-section area**
  - **Material**



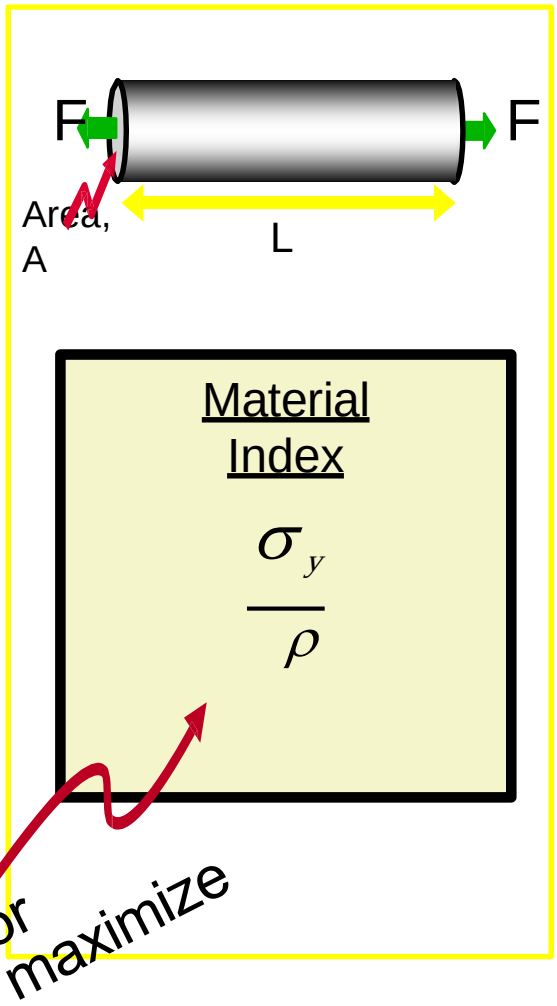
- **Objective:**
  - $m = AL\rho$
- **Constraint:**
  - $F/A < \sigma_y$

- **Objective:**
  - $m = AL\rho$
- **Constraint:**
  - $F/A < \sigma_y$
- **Rearrange to eliminate free variable**

$$m \geq (F)(L)\left(\frac{\rho}{\sigma_y}\right)$$

- **Minimize weight by minimizing**

$$\left(\frac{\sigma_y}{\rho}\right)$$

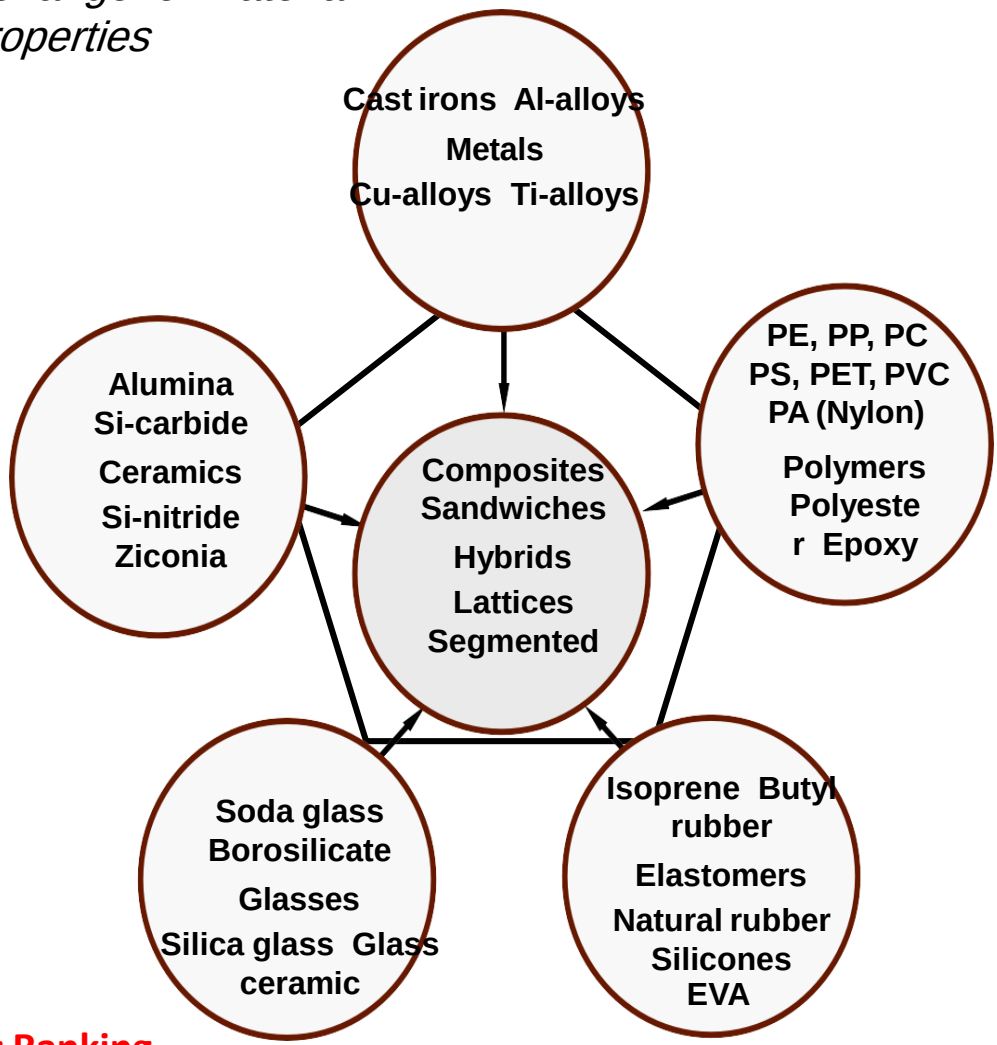


Second Step: Screening

"Eliminate materials that cannot do the job"

Need effective way of

evaluating large range of material classes and properties



Third Step: Ranking

"Find the materials that do the job best"

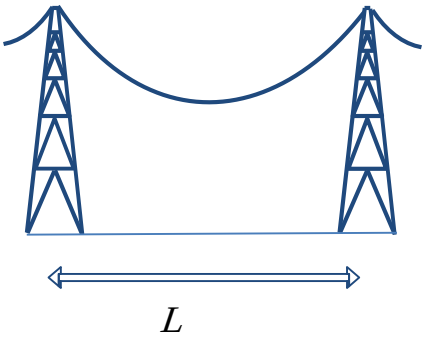
What if multiple materials are selected after screening?

Which one is best?

What if there are multiple material parameters for evaluation?

Use Material Index

Single Property Ranking Example: Overhead Transmission Cable



Q **Function:**

- **Transmit electricity**

Q **Objective:**

- **Minimize electrical Resistance**

Q **Constraints:**

$$R = \rho_e \frac{L}{A}$$

Electrical resistivity

- **Length L and section A are specified**
- **Must not fail under wind or ice-load so the required tensile strength > 80 MPa**

Q **Free variables:**

- **Material choice**

Screen on strength, rank on resistivity

Advanced Ranking: The Material Index

The method

1. Identify function, constraints, objective and free variables

- **List simple constraints for screening**

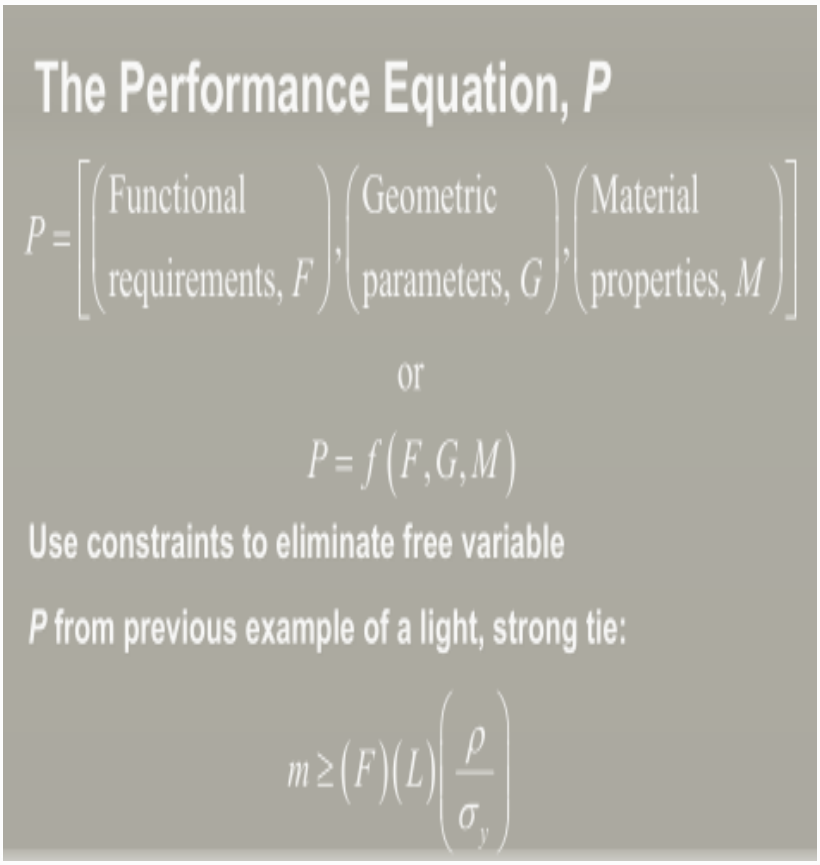
2. Write down equation for objective -- the "performance equation"

- **If objective involves a free variable other than the material)**
  - ***Identify the constraint that limits it***
  - ***Use this to eliminate the free variable in performance equation***

3. Read off the combination of material properties that maximizes performance -- the material index

- **Use this for ranking**

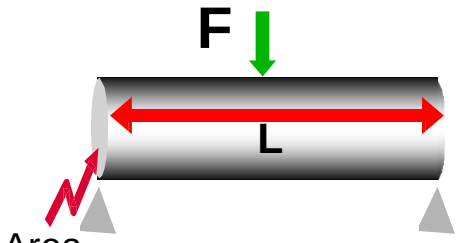
4.



The Material Index

Example: Materials for a stiff, light beam

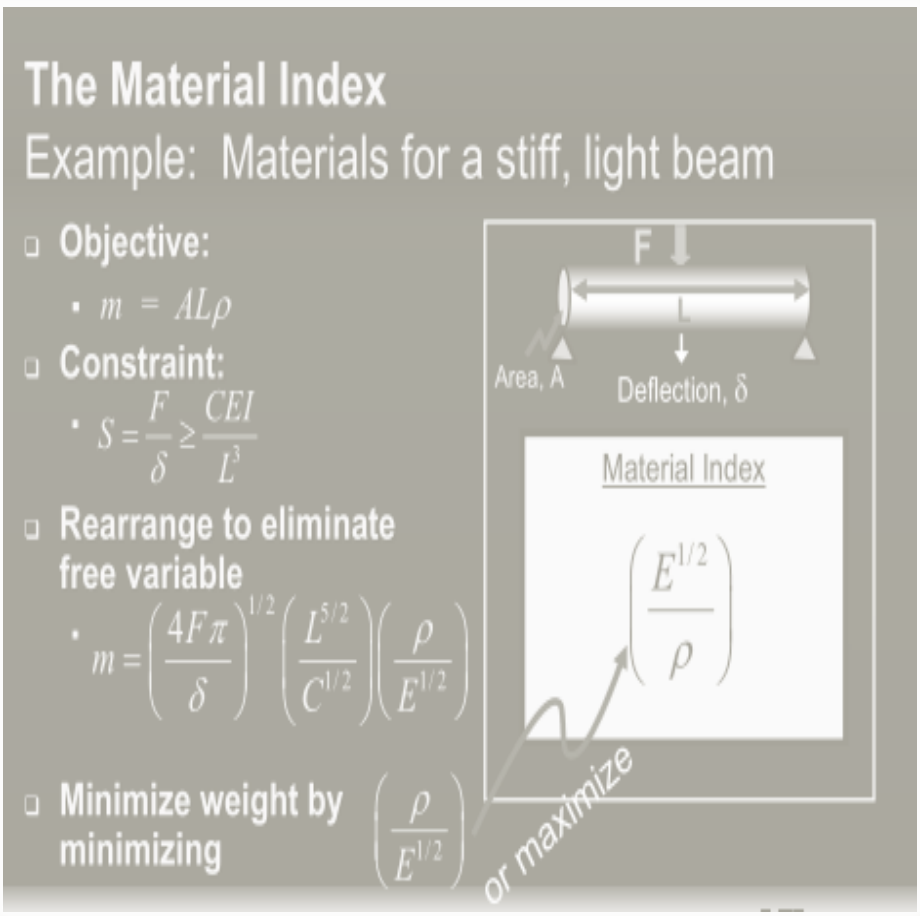
- Q **Function:**
  - **Support a bending load**
- Q **Objective:**
  - **Minimize mass**
- Q **Constraints:**
  - **Length specified**
  - **Carry load F, without too much deflection**
- Q **Free variables:**
  - **Cross-section area**
  - **Material**



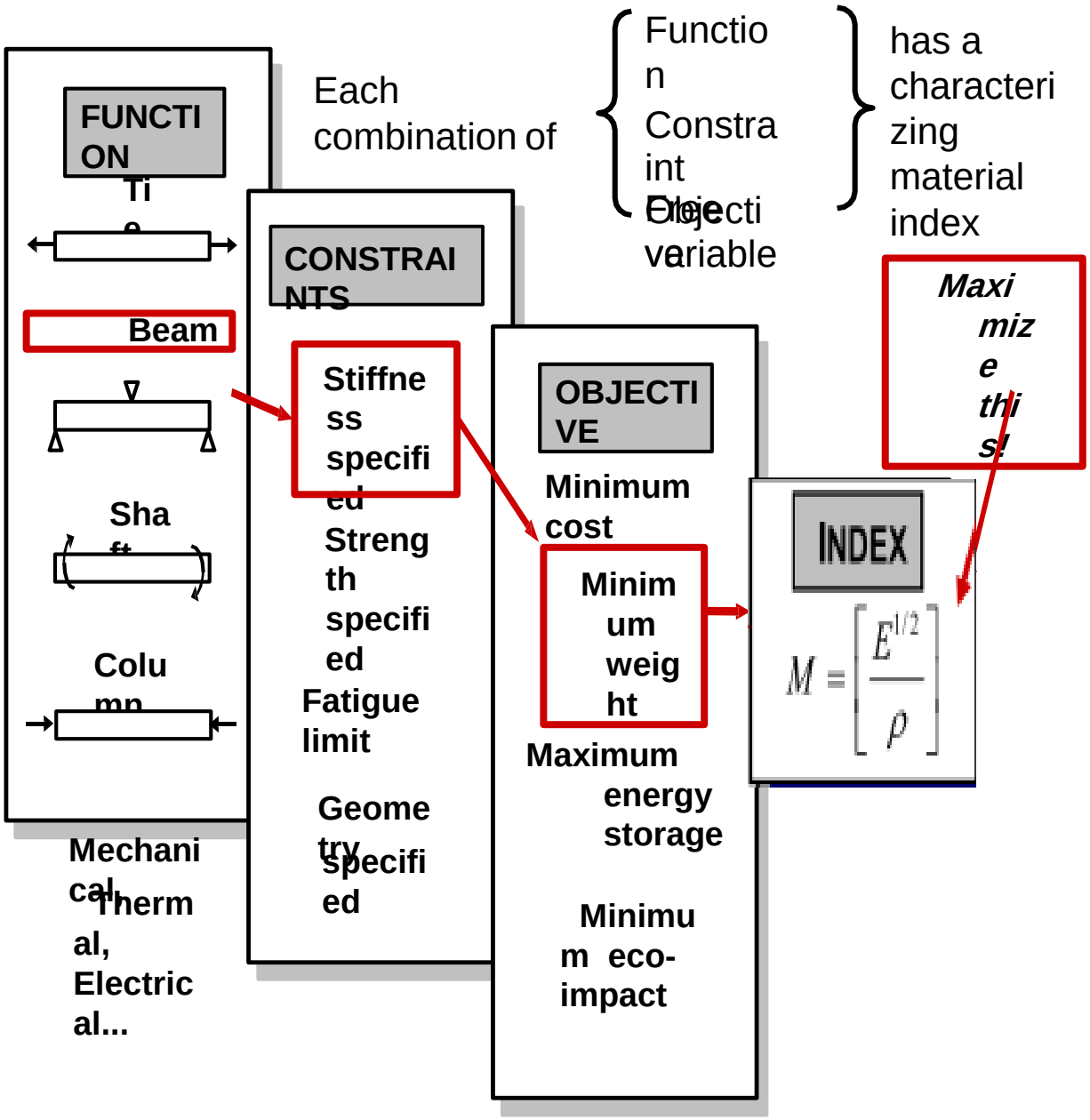
Area, A

Deflection, δ

- Q **Objective:**
  - $m = AL\rho$
- Q **Constraint:**
  - $S = \frac{F}{\delta} \geq \frac{CEI}{L^3}$



Material Index Calculation Process Flow



Material Index Examples

Q **An objective defines a performance metric: e.g. mass or resistance**

Q **The equation for performance metric contains material properties**

Q **Sometimes a single property**

Q **Sometimes a combination**

Either is a Material

Loading	Stiffness Limited	Strength Limited
Tension	E/ρ	σ_f/ρ
Bending	$E^{1/2}/\rho$	$\sigma_f^{2/3}/\rho$
Torsion	$G^{1/2}/\rho$	$\sigma_f^{2/3}/\rho$

- Material Indices for a Beam

- Objective: Minimize Mass

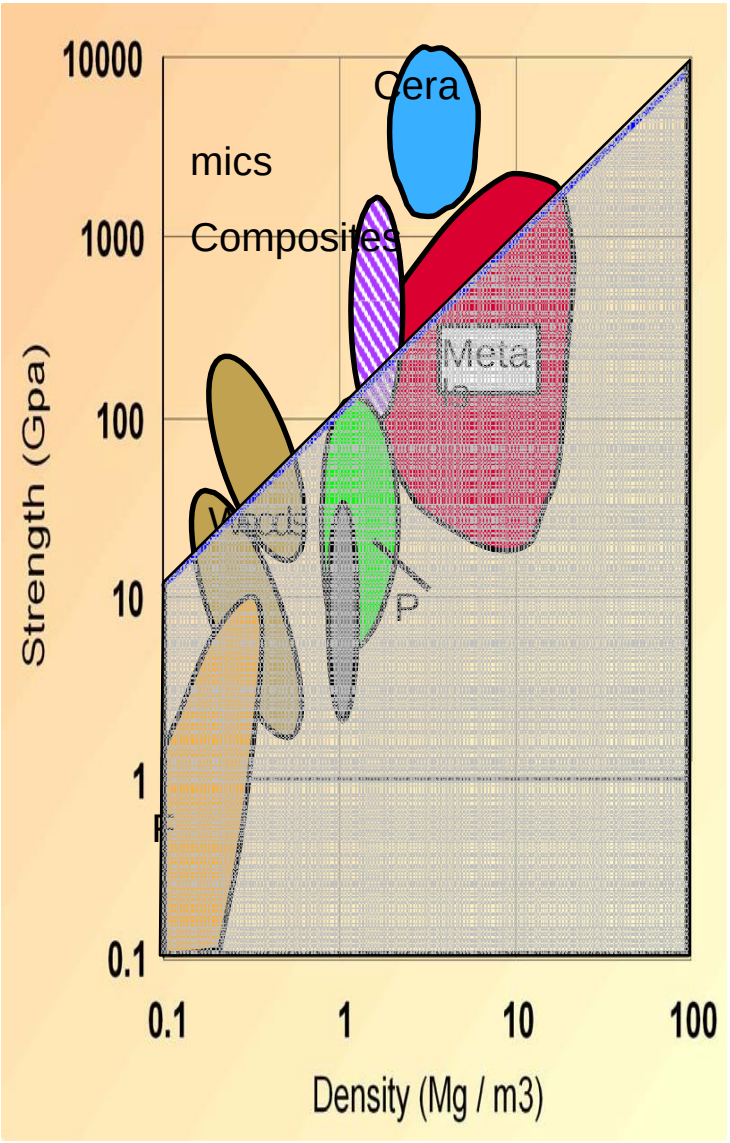
- Performance Metric: Mass

- *aximize!*

Optimized Selection Using Material Indices & Property Charts:

Strength

- Example: Tension Load, strength limited
- Q Maximize: $M = \sigma / \rho$
- Q In log space:
- $\log \sigma = \log \rho + \log M$
- Q This is a set of lines with slope=1
- Q Materials above line are candidates



3-THIN AND THICK CYLINDERS

INTRODUCTION:

In many engineering applications, cylinders are frequently used for transporting or storing of liquids, gases or fluids.

Eg: Pipes, Boilers, storage tanks etc.

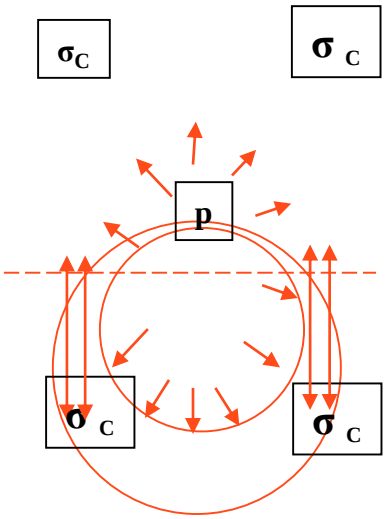
These cylinders are subjected to fluid pressures. When a cylinder is subjected to a internal pressure, at any point on the cylinder wall, three types of stresses are induced on three mutually perpendicular planes.

They are,

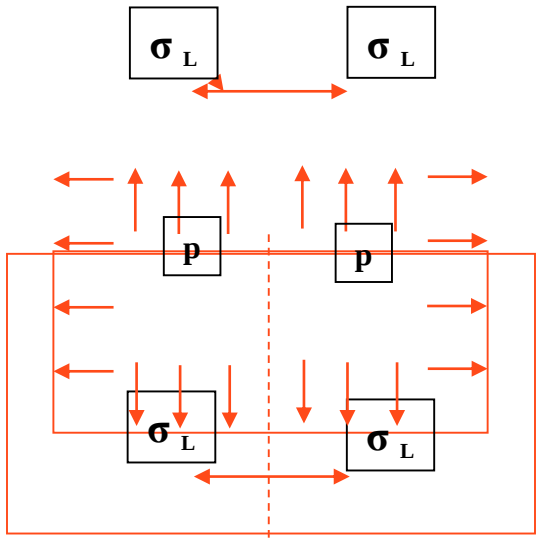
1. Hoop or Circumferential Stress (σ_C) – This is directed along the tangent to the circumference and tensile in nature. Thus, there will be increase in diameter.

2. Longitudinal Stress (σ_L) – This stress is directed along the length of the cylinder. This is also tensile in nature and tends to increase the length.

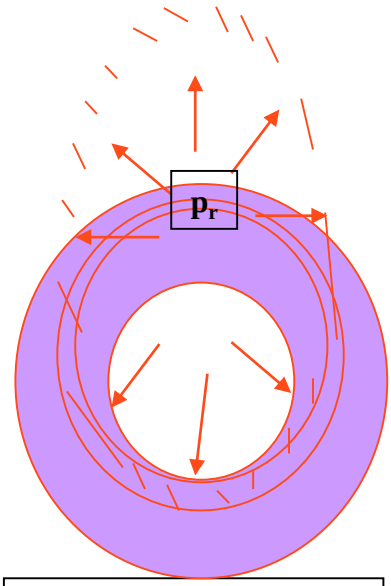
3. Radial pressure (p_r) – It is compressive in nature. Its magnitude is equal to fluid pressure on the inside wall and zero on the outer wall if it is open to atmosphere.



1. Hoop Stress (σ_C)



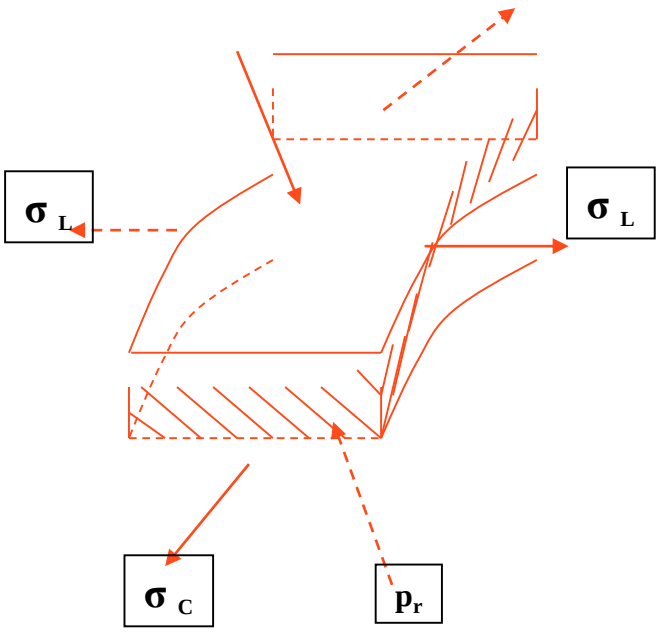
2. Longitudinal Stress (σ_L)



3. Radial Stress (p_r)



Element on the cylinder wall subjected to these three stresses



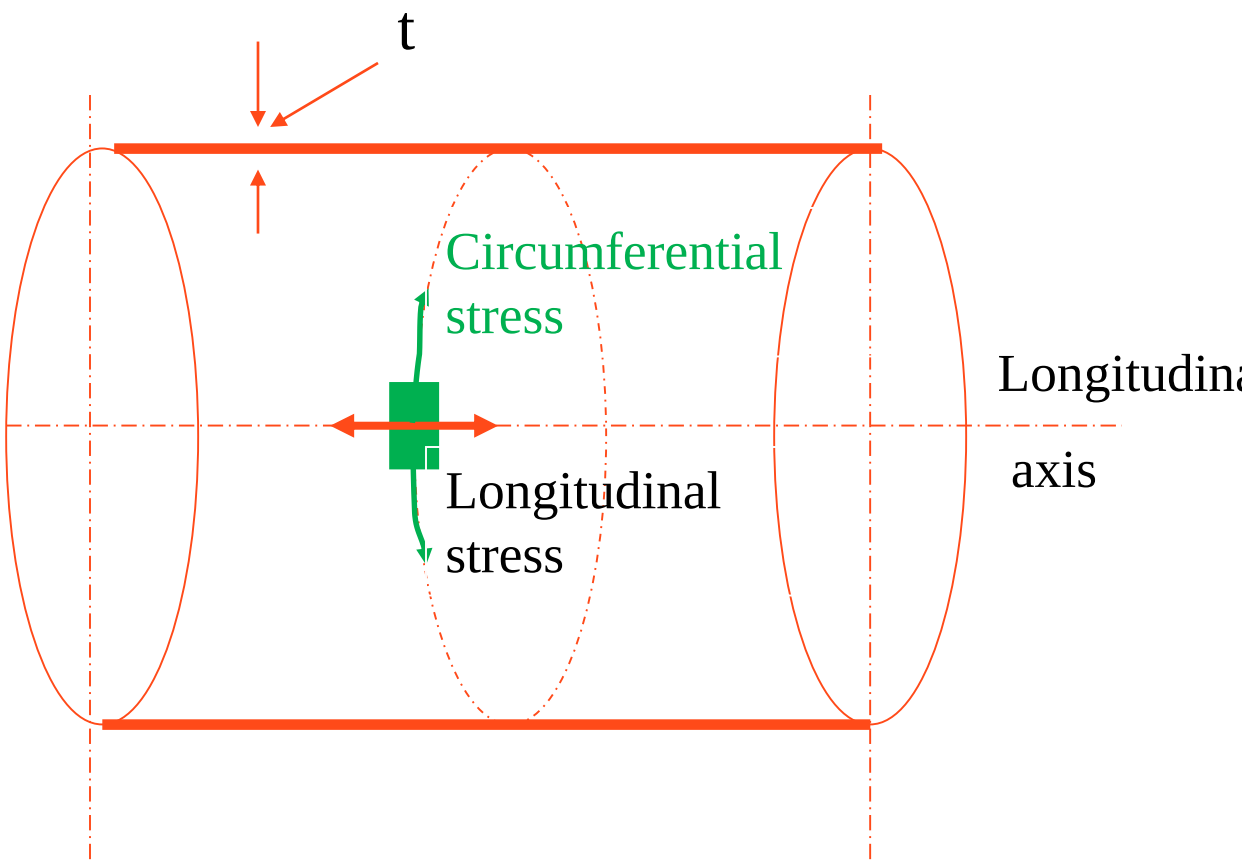
THIN CYLINDERS

INTRODUCTION:

A cylinder or spherical shell is considered to be thin when the metal thickness is small compared to internal diameter.

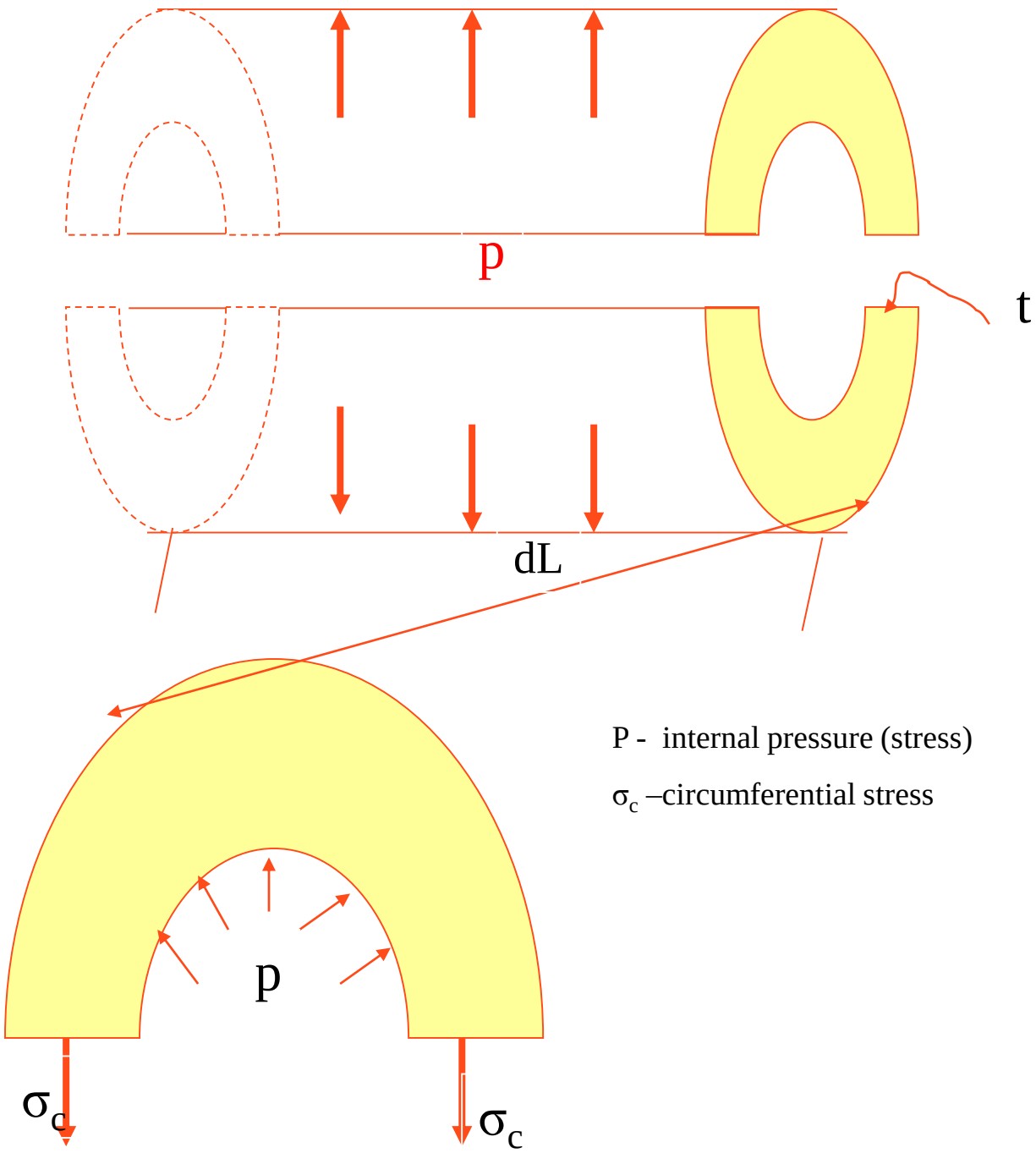
i. e., when the wall thickness, ‘t’ is equal to or less than ‘d/20’, where ‘d’ is the internal diameter of the cylinder or shell, we consider the cylinder or shell to be thin, otherwise thick.

Magnitude of radial pressure is very small compared to other two stresses in case of thin cylinders and hence neglected.



The stress acting along the circumference of the cylinder is called circumferential stresses whereas the stress acting along the length of the cylinder (i.e., in the longitudinal direction) is known as longitudinal stress

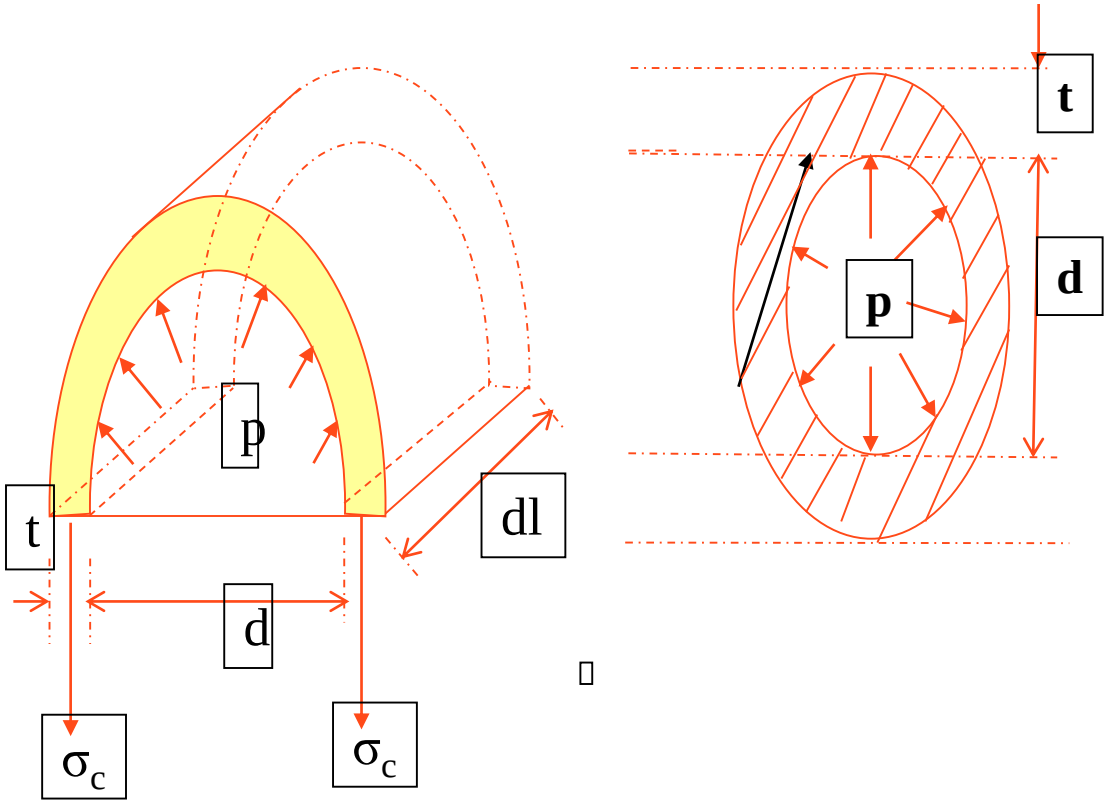
The bursting will take place if the force due to internal (fluid) pressure (acting vertically upwards and downwards) is more than the resisting force due to circumferential stress set up in the material.



P - internal pressure (stress)

σ_c –circumferential stress

EVALUATION OF CIRCUMFERENTIAL or HOOP STRESS (σ_c):



Consider a thin cylinder closed at both ends and subjected to internal pressure ‘p’ as shown in the figure.

Let d=Internal diameter, t = Thickness of the wall

L = Length of the cylinder.

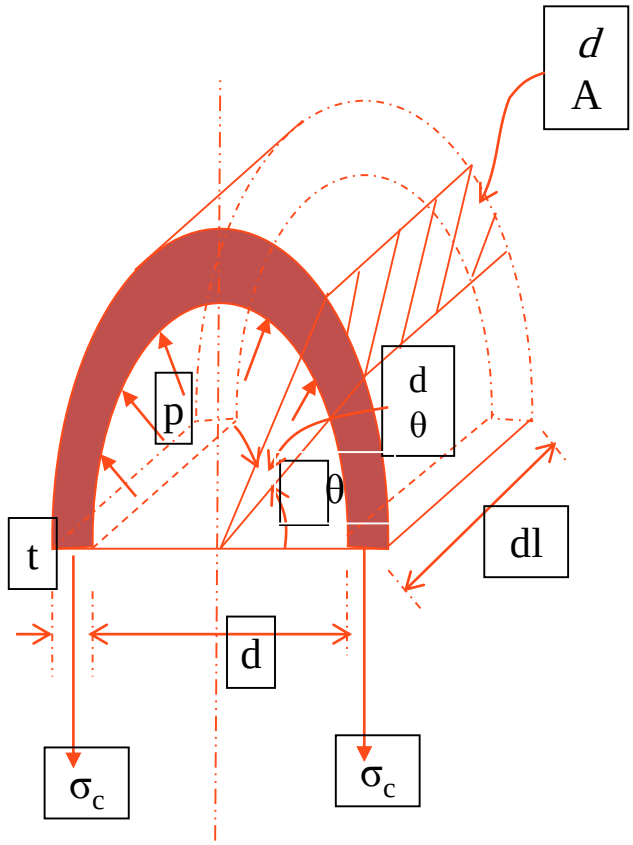
To determine the Bursting force across the diameter:

Consider a small length '*dl*' of the cylinder and an elementary area '*dA*' as shown in the figure.

Force on the elementary area,

$$dF = \mathrm{p} \times dA$$
$$= \mathrm{p} \times \mathrm{r} \times dl \times d\theta$$

$$= \mathrm{p} \times \frac{\mathrm{d}}{2} \times dl \times d\theta$$

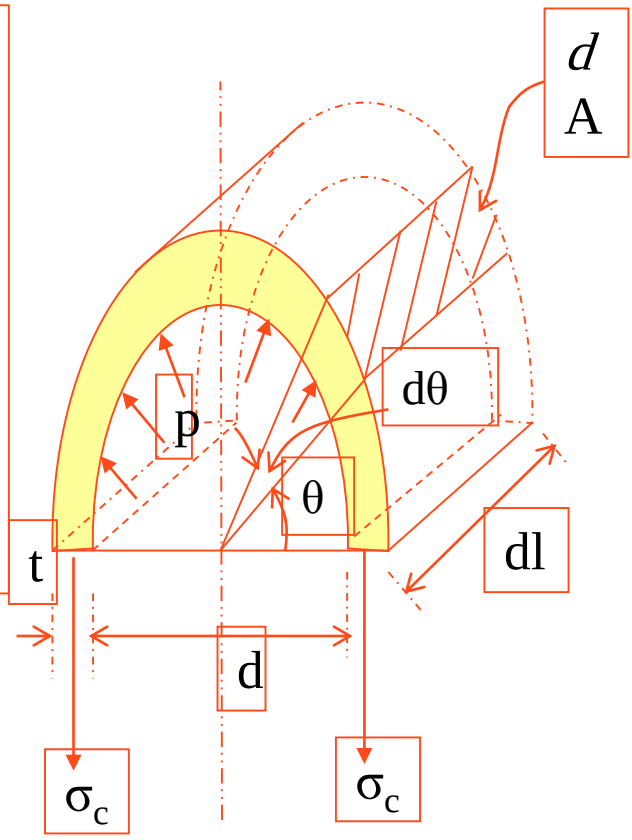


Horizontal component of this force

$$dF_x$$
$$= \mathrm{p} \times \frac{\mathrm{d}}{2} \times dl \times \cos\theta$$
$$\times d\theta$$

Vertical component of this force

The horizontal components cancel out when integrated over semi-circular portion as there will be another equal and opposite horizontal component on the other side of the vertical axis.



∴ Total diametrical bursting force

$$= \int_0^{\pi} \mathrm{p} \times \frac{\mathrm{d}}{2} \times dl \times \sin\theta \times d\theta$$

$$= \mathrm{p} \times \frac{\mathrm{d}}{2} \times \mathrm{dl} \times [-\cos\theta]_0^{\pi} = \underline{\mathrm{p} \times \mathrm{d} \times dl}$$

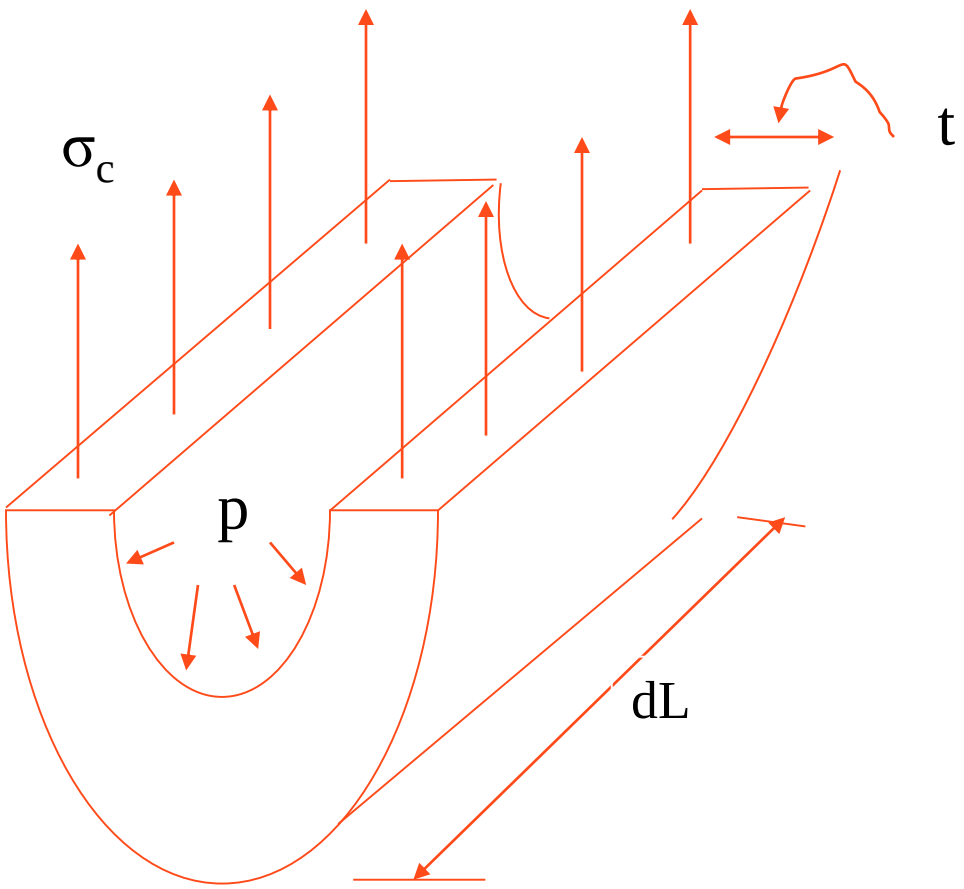
$$= \mathrm{p} \times \text{projected area of the curved surface.}$$

$\therefore$ Resisting force (due to circumferential stress σ_c)
$= 2 \times \sigma_c \times t \times dl$

Under equillibrium, Resisting force
= Bursting force

$= p \times d \times dl$

$\therefore$ Circumferential stress, $\underline{\sigma_c = \frac{p \times d}{2 \times t}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$



Assumed as rectangular

Force due to fluid pressure = p × area on which p is acting = p ×(d ×L)

(bursting force)

Force due to circumferential stress = σ_c × area on which σ_c is acting

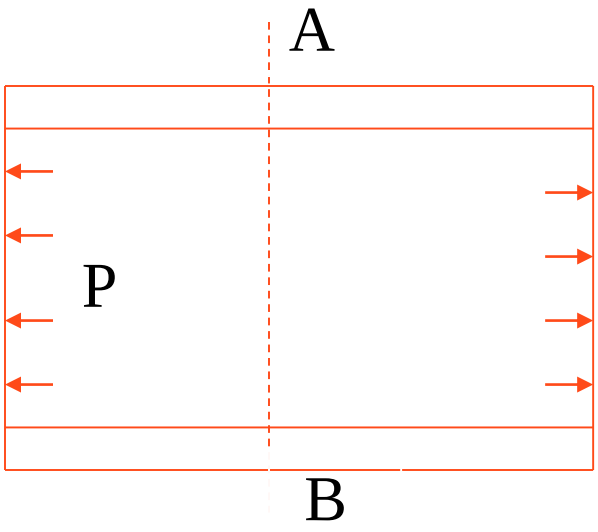
(resisting force) = $\sigma_c \times (L \times t + L \times t) = \sigma_c \times 2 L \times t$

Under equilibrium bursting force = resisting force

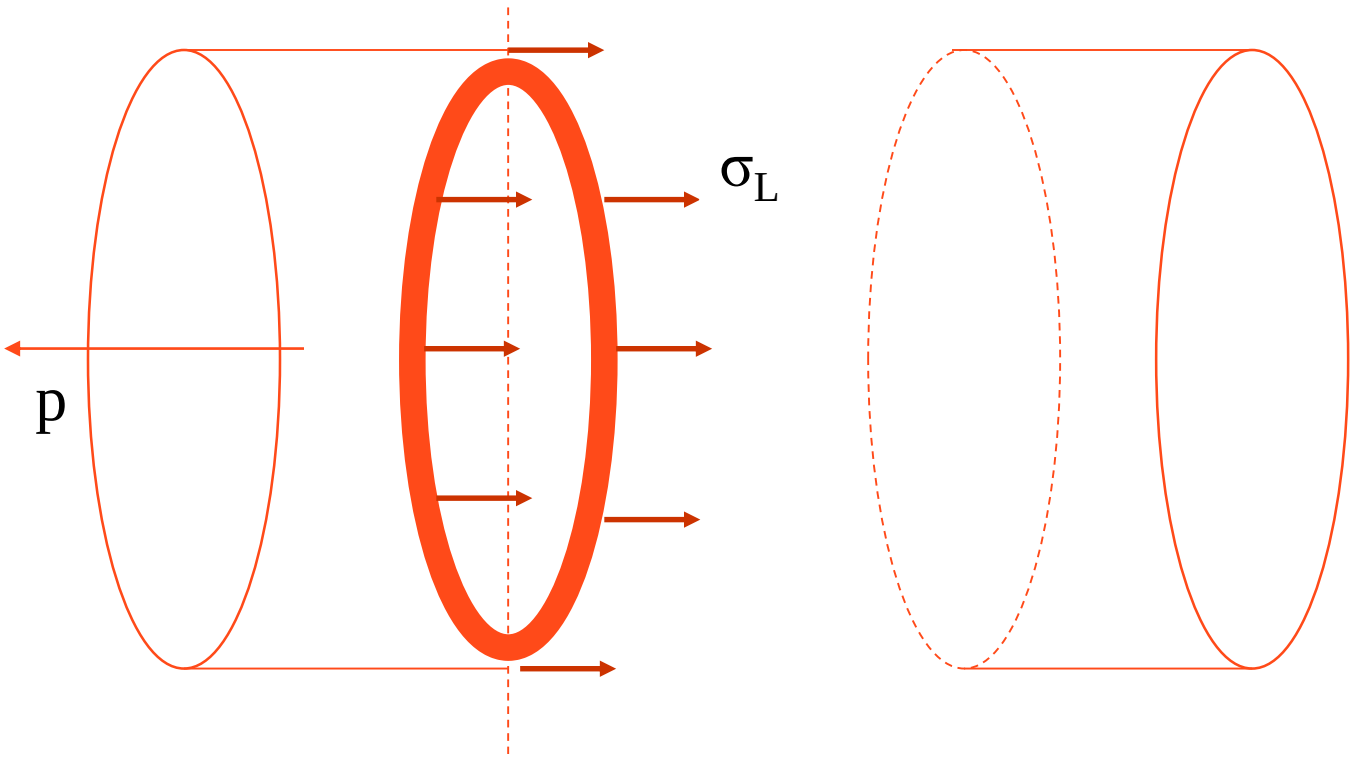
$$p \times (d \times L) = \sigma_c \times 2 L \times t$$

∴ Circumferential stress, $\sigma_c = \dfrac{p \times d}{2 \times t}$ ..... (1)

LONGITUDINAL STRESS (σ_L):

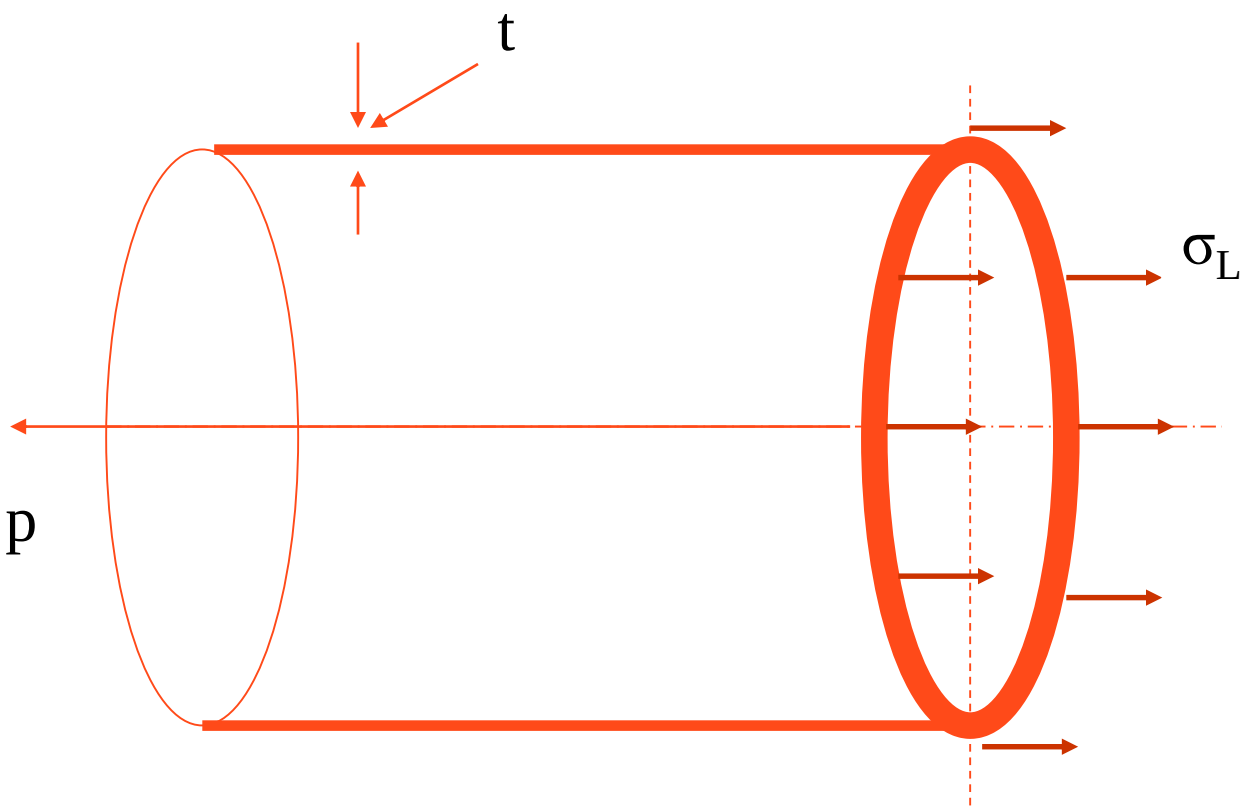


The bursting of the cylinder takes place along the section AB



The force, due to pressure of the fluid, acting at the ends of the thin cylinder, tends to burst the cylinder as shown in figure

EVALUATION OF LONGITUDINAL STRESS (σ_L):



Longitudinal bursting force (on the end of cylinder)

$= p \times \frac{\pi}{4} \times d^2$

Area of cross section resisting this force $= \pi \times d \times t$

Let σ_L

= Longitudinal stress of the material of the cylinder.

Under equillibrium, bursting force
$=$ resisting force

$$\text{i.e., } p \times \frac{\pi}{4} \times d^2$$

$$= \sigma_L \times \pi \times d \times t$$

$$\therefore \quad \text{Longitudinal stress, } \sigma_L = \frac{p \times d}{4 \times t} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From eqs (1) & (2), $\underline{\sigma_C = 2 \times \sigma_L}$

Force due to fluid pressure
$= p \times$ area on which p is acting

$$= \mathrm{p} \times \frac{\pi}{4} \times \mathrm{d}^2$$

Re $s\ isting force = \sigma_L \times area on which \sigma_L is acting$
$= \sigma_L \times \pi \times \mathrm{d} \times \mathrm{t}$

circumference



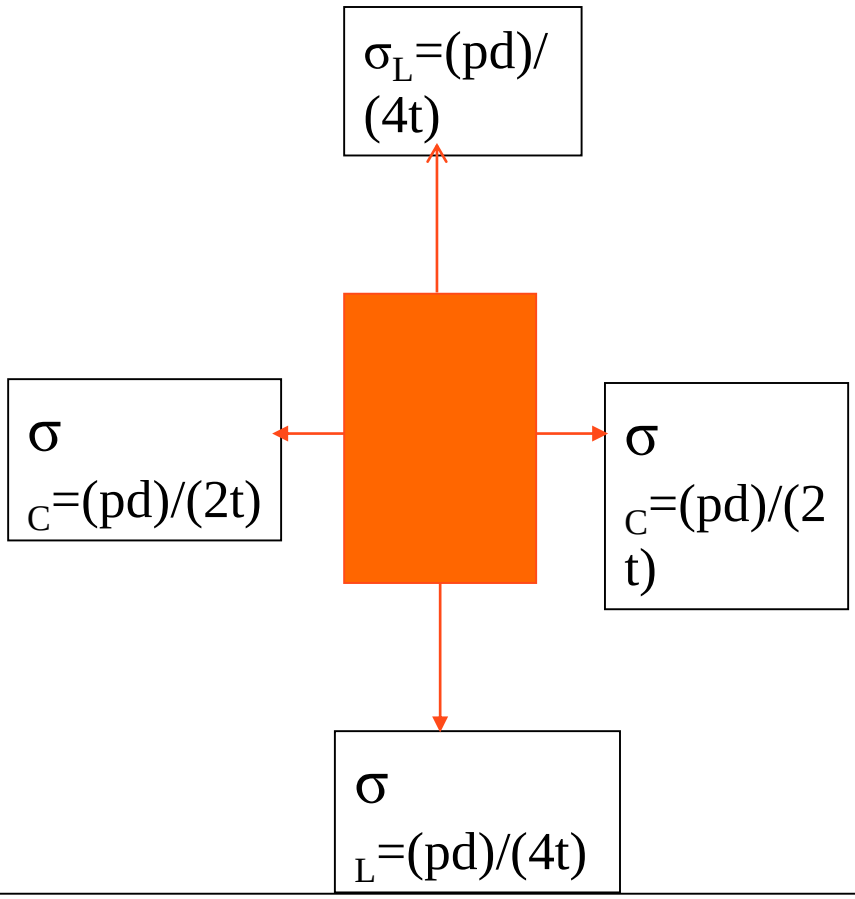
Under equillibrium, bursting force
$=$ resisting force

$$\text{i.e., } \mathrm{p} \times \frac{\pi}{4} \times \mathrm{d}^2 = \sigma_L \times \pi \times \mathrm{d} \times \mathrm{t}$$

$\therefore$ Longitudinal stress, σ_L

$$= \frac{\mathrm{p} \times \mathrm{d}}{4 \times \mathrm{t}} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

EVALUATION OF STRAINS



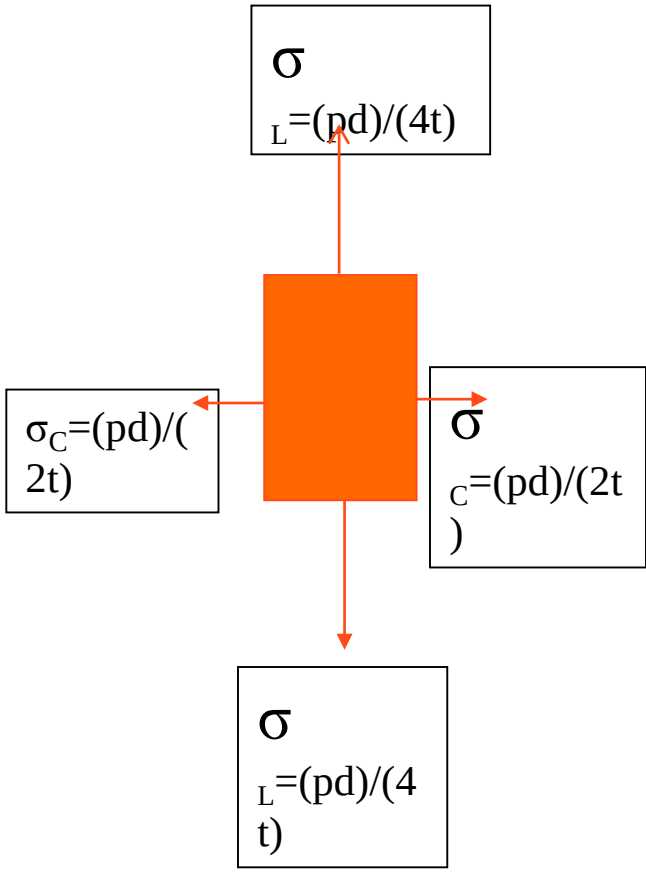
A point on the surface of thin cylinder is subjected to biaxial stress system, (Hoop stress and Longitudinal stress) mutually perpendicular to each other, as shown in the figure. The strains due to these stresses i.e., circumferential and longitudinal are obtained by applying Hooke's law and Poisson's theory for elastic materials.

Circumferential strain, ε_C:

$$\varepsilon_C = \frac{\sigma_C}{E} - \mu \times \frac{\sigma_L}{E}$$

$$= 2 \times \frac{\sigma_L}{E} - \mu \times \frac{\sigma_L}{E}$$

$$= \frac{\sigma_L}{E} \times (2 - \mu)$$



i.e.,

$$\varepsilon_C = \frac{\delta d}{d} = \frac{p \times d}{4 \times t \times E} \times (2 - \mu) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Note: Let δd be the change in diameter. Then

$$\varepsilon_c = \frac{final\ circumference - original\ circumference}{original\ circumference}$$

$$= \left[\frac{\pi(d + \delta d) - \pi d}{\pi d}\right] = \frac{\delta d}{d}$$

Longitudinal strain, ε_L:

$$\varepsilon_L = \frac{\sigma_L}{E} - \mu \times \frac{\sigma_C}{E}$$

$$= \frac{\sigma_L}{E} - \mu \times \frac{(2 \times \sigma_L)}{E}$$

$$= \frac{\sigma_L}{E} \times (1 - 2 \times \mu)$$

i.e.,

$$\varepsilon_L = \frac{\delta l}{L} = \frac{p \times d}{4 \times t \times E} \times (1 - 2 \times \mu) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

VOLUMETRIC STRAIN, $\frac{\delta v}{V}$

Change in volume = δV = final volume – original volume

original volume = V = area of cylindrical shell × length

$$= \frac{\pi d^2}{4} L$$

final volume = final area of cross section × final length

$$= \frac{\pi}{4}[d + \delta d]^2 \times [L + \delta L]$$
$$= \frac{\pi}{4}[d^2 + (\delta d)^2 + 2d\delta d] \times [L + \delta L]$$
$$= \frac{\pi}{4}[d^2 L + (\delta d)^2 L + 2Ld\delta d + d^2\delta L + (\delta d)^2\delta L + 2d\delta d\delta L]$$

$$neglecting the smaller quantities such as (\delta d)^2 L, (\delta d)^2 \delta L and 2d\delta d\delta L$$
$$Final volume = \frac{\pi}{4}[d^2 L + 2Ld\delta d + d^2\delta L]$$

$$change in volume \delta V = \frac{\pi}{4}[d^2 L + 2Ld\delta d + d^2\delta L] - \frac{\pi}{4}[d]^2 L$$
$$\delta V = \frac{\pi}{4}[2Ld\delta d + d^2\delta L]$$

$$\frac{\mathrm{dv}}{\mathrm{V}} = \frac{\frac{\pi}{4}[2dL\delta d + \delta L d^2]}{\frac{\pi}{4} \times \mathrm{d}^2 \times \mathrm{L}}$$

$$= \frac{\delta \mathrm{L}}{\mathrm{L}} + 2 \times \frac{\delta \mathrm{d}}{\mathrm{d}}$$

$$\frac{\mathrm{dV}}{\mathrm{V}} = \varepsilon_{\mathrm{L}} + 2 \times \varepsilon_{\mathrm{C}}$$

$$= \frac{\mathrm{p} \times \mathrm{d}}{4 \times \mathrm{t} \times \mathrm{E}}(1 - 2 \times \mu) + 2 \times \frac{\mathrm{p} \times \mathrm{d}}{4 \times \mathrm{t} \times \mathrm{E}}(2 - \mu)$$

i.e.,

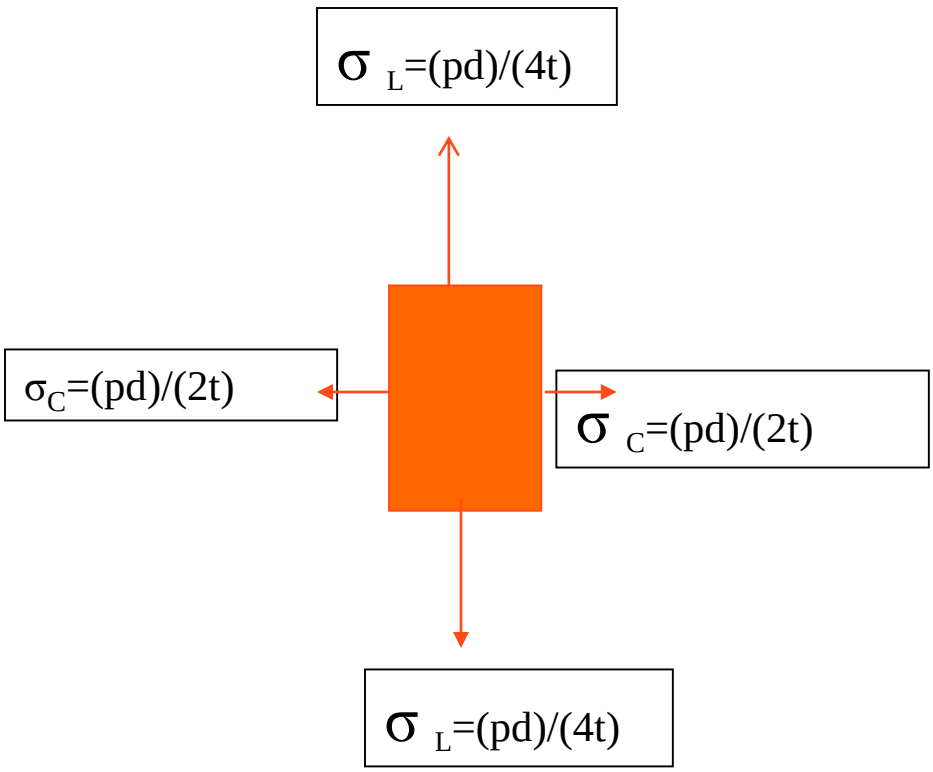
$$\underline{\frac{\mathrm{dv}}{\mathrm{V}} = \frac{\mathrm{p} \times \mathrm{d}}{4 \times \mathrm{t} \times \mathrm{E}}(5 - 4 \times \mu)} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Maximum Shear stress:

There are two principal stresses at any point, viz., Circumferential and longitudinal. Both these stresses are normal and act perpendicular to each other.

$\therefore$ Maximum Shear stress, $\tau_{max} = \frac{\sigma_C - \sigma_L}{2}$

$$= \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2}$$



i.e.,

$$\tau_{max} = \frac{pd}{8t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Maximum Shear stress:

$\therefore$ Maximum Shear stress, τ_{max}

$$= \frac{\sigma_C - \sigma_L}{2}$$

$$= \frac{\frac{pd}{2t} - \frac{pd}{4t}}{2}$$

i.e.,

$$\tau_{max} = \frac{pd}{8t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Application of finite element method

Finite Element Analysis (FEA)

FEA is originally developed for solving solid mechanics problem.

Solid Mechanics:



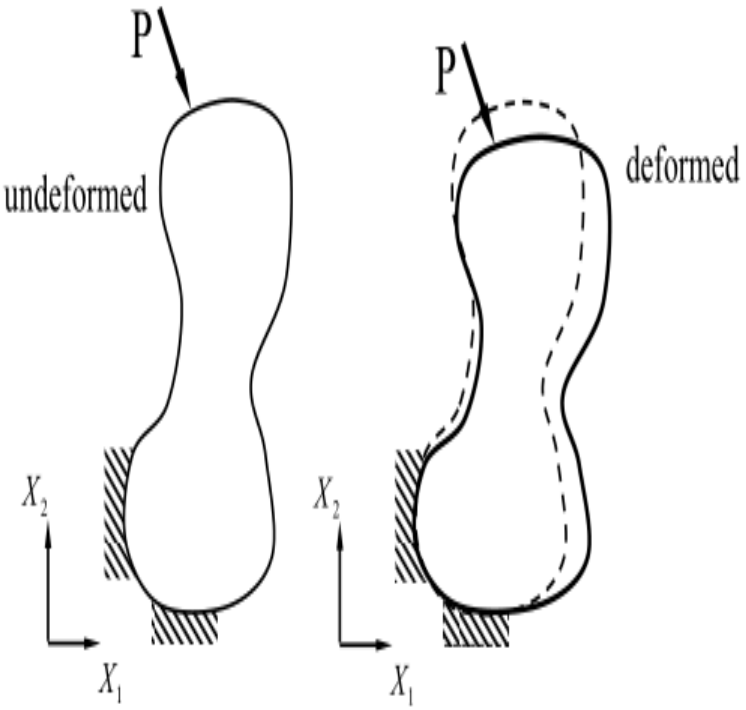
Fig.1

Object: A Solid with known mechanical properties. (a shaft; human tissue……)

Concepts:

- Boundary: The surface enclosing the geometry
- Solid: Interior + Boundary
- Boundary conditions: Any prescribed quantities, such as prescribed displacements and prescribed tractions on the boundary

Solid Mechanics:



Question:
If we apply a force on a solid, what are the values of the displacements, stresses, and strains at **EACH MATERIAL POINT**?

Input

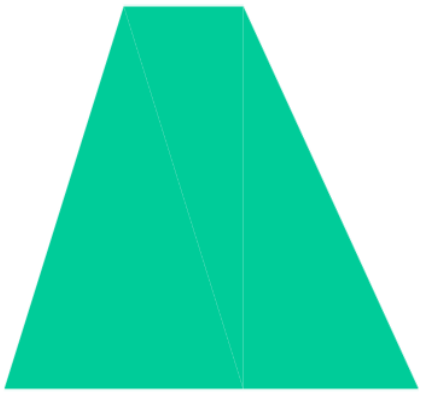
Boundary conditions (prescribed force; prescribed displacement …)

Output

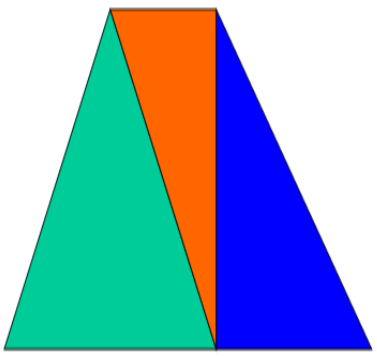
Stresses, strains, displacements, at each material point (X_1,X_2,X_3)

? ? ?

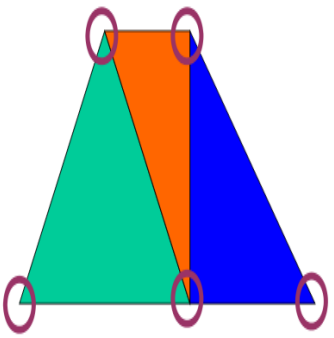
Object



Elements



Nodes



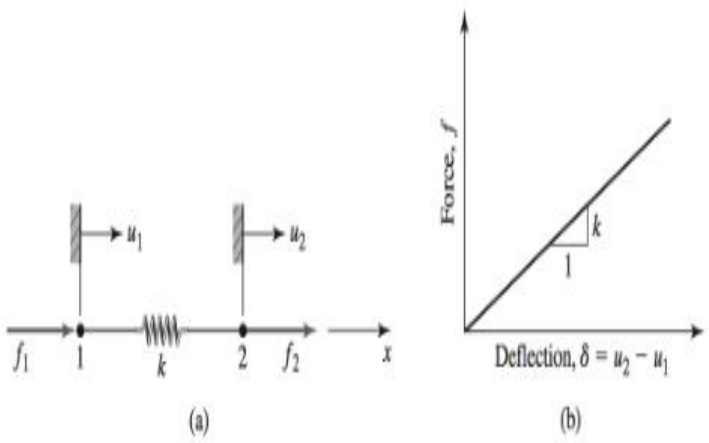
Displacement → Stress

→ Strain

LINEAR SPRING AS A FINITE ELEMENT

A linear elastic spring is a mechanical device capable of supporting axial loading only, and the elongation or contraction of the spring is directly proportional to the applied axial load. The constant of proportionality between deformation and load is referred to as the spring constant, spring rate, or spring stiffness k, and has units of force per unit length.

As an elastic spring supports axial loading only, we select an element coordinate system (also known as a local coordinate system) as an x axis oriented along the length of the spring, as shown.



(a) Linear spring element with nodes, nodal displacements, and nodal forces.
(b) Load-deflection curve.

Assuming that both the nodal displacements are zero when the spring is undeformed, the net spring deformation is given by

$\delta = u_2 - u_1$

and the resultant axial force in the spring is

$f = k\delta = k(u_2 - u_1)$

For equilibrium,

$f_1 + f_2 = 0$ or $f_1 = -f_2$,

Then, in terms of the applied nodal forces as

$$f_1 = -k(u_2 - u_1)$$

$$f_2 = k(u_2 - u_1)$$

which can be expressed in matrix form as

$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} \quad \text{or} \quad [k_e]\{u\} = \{f\}$$

where

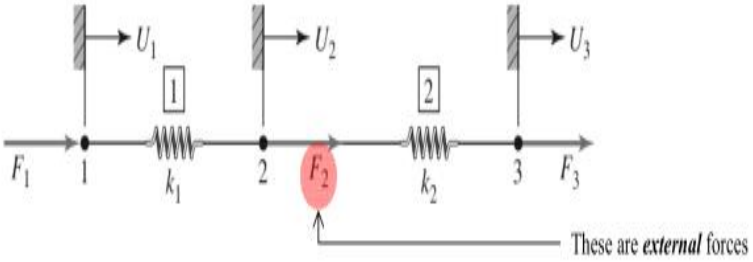
$$[k_e] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$ **Stiffness matrix for one spring element**

is defined as the element stiffness matrix in the element coordinate system (or local system), $\{u\}$ is the column matrix (vector) of nodal displacements, and $\{f\}$ is the column matrix (vector) of element nodal forces.

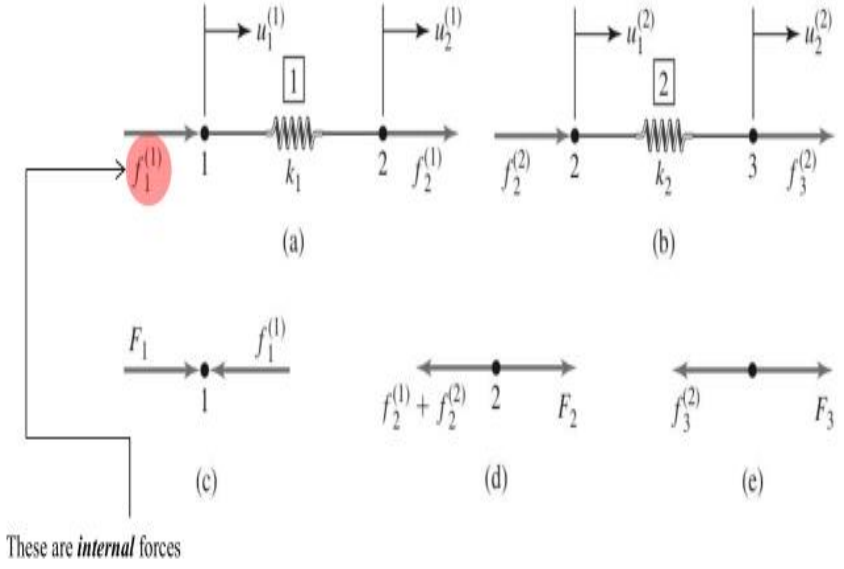
$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = [k_e] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \quad \text{with} \quad [k_e] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

known $\{F\} = [K]\{X\}$ unknown

SYSTEM OF TWO SPRINGS



Free body diagrams:



Writing the equations for each spring in matrix form:

Superscript refers to element

$$\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1^{(1)} \\ u_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \end{Bmatrix}$$

$$\begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1^{(2)} \\ u_2^{(2)} \end{Bmatrix} = \begin{Bmatrix} f_2^{(2)} \\ f_3^{(2)} \end{Bmatrix}$$

To begin assembling the equilibrium equations describing the behavior of the system of two springs, the displacement ***compatibility conditions***, which relate element displacements to system displacements, are written as:

$$u_1^{(1)} = U_1 \qquad u_2^{(1)} = U_2 \qquad u_1^{(2)} = U_2 \qquad u_2^{(2)} = U_3$$

And
therefore:

$$\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \end{Bmatrix}$$

$$\begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} f_2^{(2)} \\ f_3^{(2)} \end{Bmatrix}$$

Here, we use the notation $f^{(j)}_i$ to represent the force exerted on element j at node i.

Expand each equation in matrix form:

$$\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} 0 \\ U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ f_2^{(2)} \\ f_3^{(2)} \end{Bmatrix}$$

Summing member by member:

$$\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} f_1^{(1)} \\ f_2^{(1)} + f_2^{(2)} \\ f_3^{(2)} \end{Bmatrix}$$

Next, we refer to the free-body diagrams of each of the three nodes:

$$f_1^{(1)} = F_1 \qquad f_2^{(1)} + f_2^{(2)} = F_2 \qquad f_3^{(2)} = F_3$$

Final form:

$$\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

Where the stiffness matrix:

$$[K] = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix}$$

FEA for multiple (many) elements

$$\{F\} = [K] \cdot \{U\}$$

- $\{F\}$: Array of applied forces (one for each DOF)
- $[K]$: Matrix of stiffnesses (DOF x DOF)
- $\{U\}$: Array of displacements (one for each DOF)

$\{F\}$ is "known" (loads)

$[K]$ is "known" (geometry, material properties...elements)

$\{U\}$ is to be determined (displacements)

This can be solved mathematically using a matrix inversion method

$$\{F\} = [K] \cdot \{U\} \quad \rightarrow \quad \{U\} = [K]^{-1}\{F\}$$

CHAPTER.3
Theory of tribology and Design and selection of bearings and seals

3.1.Wear

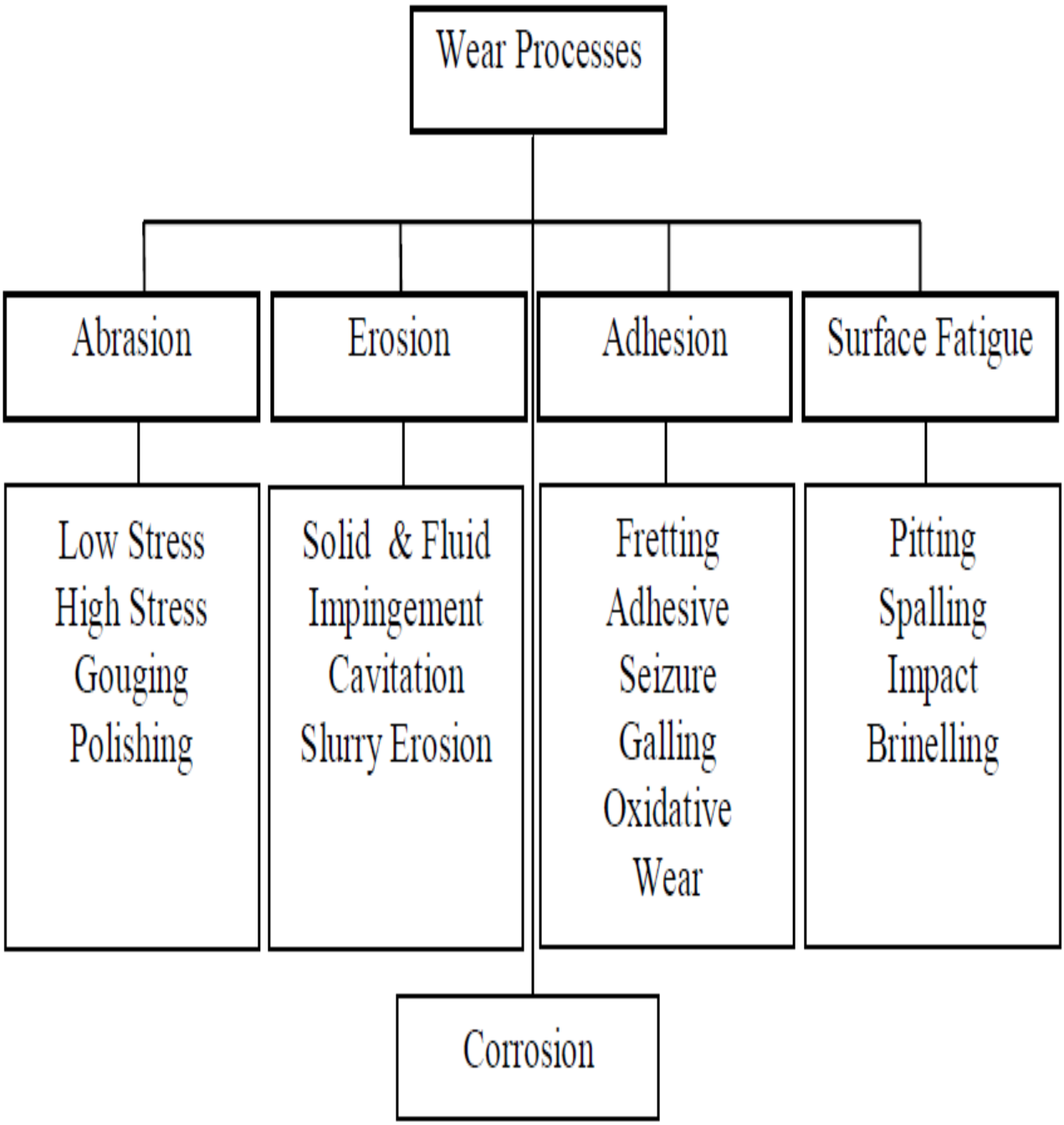
Wear is a process of removal of material from one or both of two solid surfaces in solid state contact, occurring when these two solid surfaces are in sliding or rolling motion together.

Wear is the progressive damage, involving material loss, which occurs on the surface of a component as result of its motion relative to the adjacent working parts.

Occurrence of Wear depends on

- Geometry of the surface
- Applied load
- The rolling and sliding velocities
- Environmental conditions
- Mechanical, Thermal, Chemical and Metallurgical properties
- Physical, Thermal and Chemical properties of the lubricant

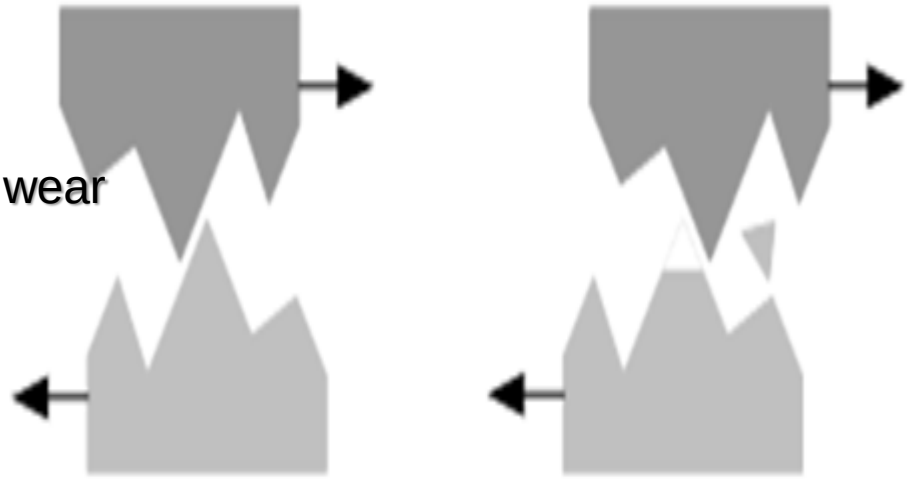
Types of wear process



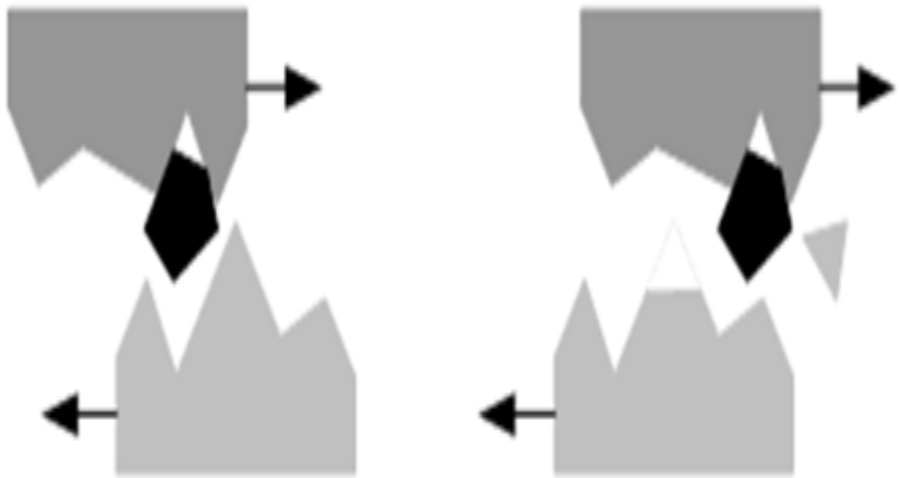
Abrasive wear

Abrasive wear occurs when a harder material is rubbing against a softer material

Two body wear



Three body wear



$$V = K\frac{NL}{3\sigma s}$$

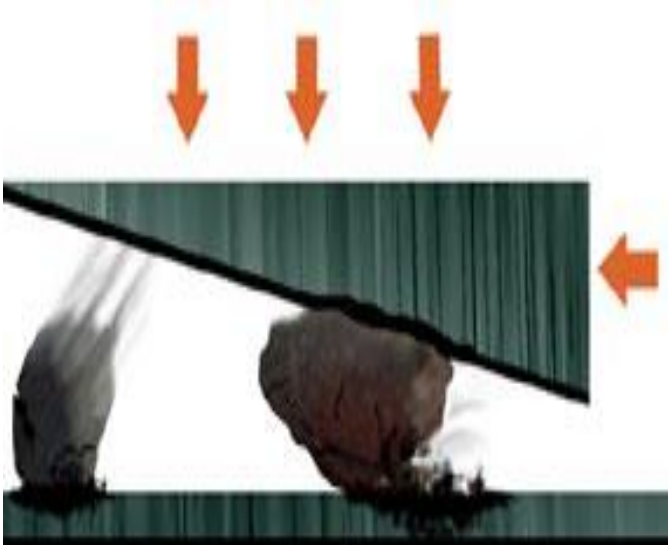
Where

V = wear volume, L = sliding velocity

N = applied load, σ_S = surface strength

K = wear coefficient

Types of abrasive wear



Gouging abrasion
- Large particles
- High compression loads

High stress or grinding abrasion
- Smaller particles
- High compression load

Low stress or scratching abrasion
- No compression load
- Scratching abrasion while material is sliding

Polishing abrasion



Erosive wear

The impingement of solid particles, or small drops of liquid or gas on the solid surface cause wear what is known as erosion of materials and components.

Pressure generated due to change in velocity

$P = \Delta V\sqrt{E\rho}$

P = Impact pressure
E = Modulus of elasticity of impacted material
ρ = Density of the fluid
V = Velocity

Types of erosion

Solid particle erosion

Surface wear by impingement of solid particles carried by a gas or fluid.

e.g. Wear of helicopter blade leading edges in dusty environments.

- **Liquid drop erosion**

  Surface wear by impingement of liquid drops.

e.g. Wear of centrifugal gas compressor blades by condensate droplets.

- **Cavitation erosion**

  Surface wear in a flowing liquid by the generation and implosive collapse of gas bubbles.

Frictional wear / adhesive wear

Two bodies sliding over or pressed into each other which promote the material transfer from one to another.

$$\frac{V}{L} = K\frac{P}{3\sigma y}$$

Where
V = wear volume
L = sliding velocity
P = applied load
σ_y = yield stress of softer material
K = wear coefficient

Surface fatigue

- Two surfaces contacting to each other under pure rolling,
- or rolling with a small amount of sliding in contact

Contact fatigue

- As one element rolls many times over the other element
- Maximum shear stress is higher than fatigue limit

For cylinder

$\tau max =$

$$0.127\sqrt{\frac{PEe}{LRe}}$$

$$Z = 0.84\sqrt{\frac{PRe}{LEe}}$$

For sphere

$\tau max =$

$$0.4\sqrt[3]{\frac{P}{R^2}Ee^2}$$

$$Z = 0.56\sqrt[3]{P\frac{Re}{Ee}}$$

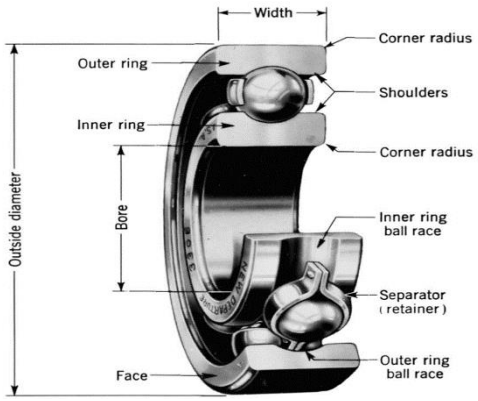
Chemical wear

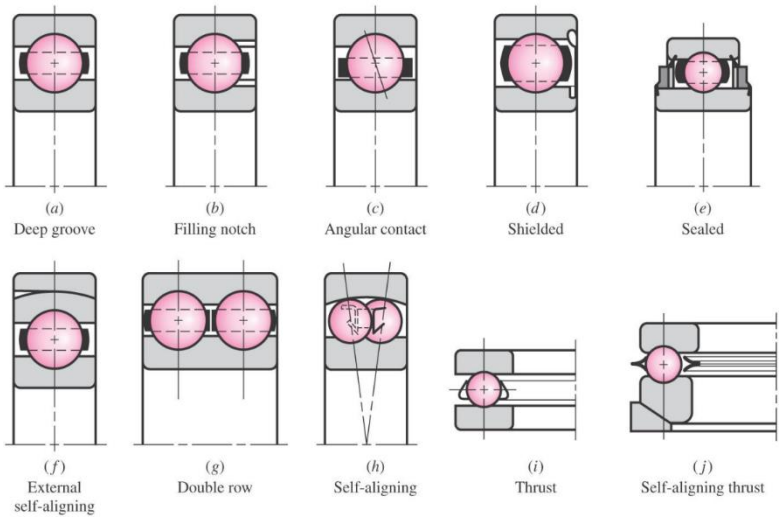
Environmental conditions produce a reaction product on one or both of rubbing
surface and this chemical product is subsequently removed by the rubbing action.

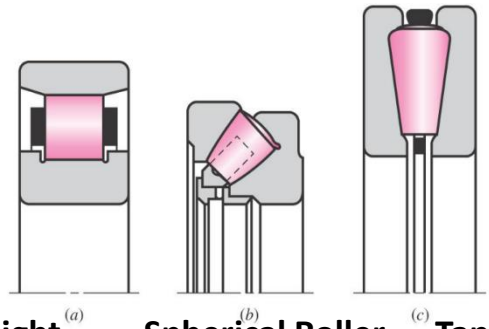
Methods for control of the wear

- Lubrication technology
- Materials substitution
- Load reduction
- Removal of impact conditions

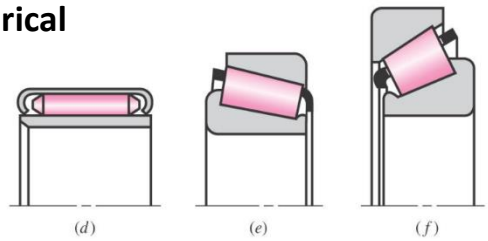
3.2.Design and selection of bearings and seals







Straight Cylindrical **Spherical Roller** **Tapered**



Needle Tapered Steep-angle

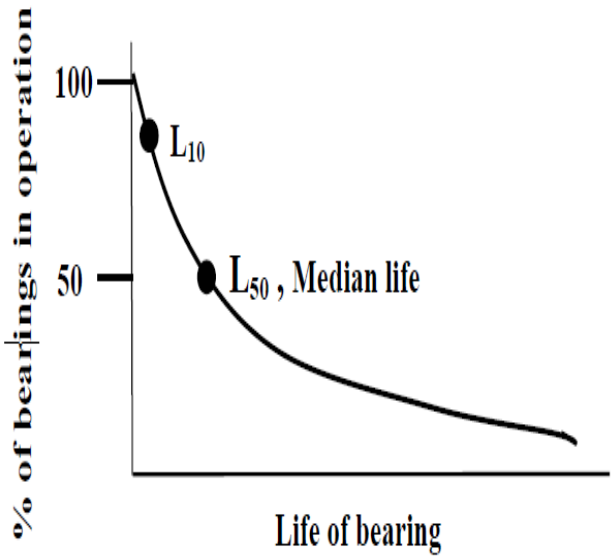
Rolling contact bearing selection

Some of the important terminologies which are required for selection of rolling contact bearing are given below.

Rating life:

Rating life is defined as the life of a group of apparently identical ball or roller bearings, in number of revolutions or hours, rotating at a given speed, so that 90% of the bearings will complete or exceed before any indication of failure occur. Suppose we consider 100 apparently identical bearings. All the 100 bearings are put onto a shaft rotating at a given speed while it is also acted upon by a load. After some time, one after another, failure of bearings will be observed. When in this process, the tenth bearing fails, then the number of revolutions or hours lapsed is recorded.

These figures recorded give the rating life of the bearings or simply L^{10} life (10 % failure). Similarly, L^{50} means, 50 % of the bearings are operational. It is known as median life. Fig. defines the life of rolling contact bearings.



Definition of life of rolling contact bearing

Bearing load

If two groups of identical bearings are tested under loads P_1 and P_2 for respective lives of L_1 and L_2, then,

$$\frac{L_1}{L_2} = \left(\frac{P_2}{P_1}\right)^a \qquad (1)$$

Where,

L : life in millions of revolution or life in hours

a : constant which is 3 for ball bearings and 10/3 for roller bearings

Basic load rating

It is that load which a group of apparently identical bearings can withstand for a rating life of one million revolutions.

Hence, if say, L_1 is taken as one million then the corresponding load is,

$$C = P(L)^{\frac{1}{a}} \qquad (2)$$

Where, C is the basic or dynamic load rating

Therefore, for a given load and a given life the value of C represents the load carrying capacity of the bearing for one million revolutions. This value of C, for the purpose of bearing selection, should be lower than that given in the manufacturer's catalogue. Normally the basic or the dynamic load rating as prescribed in the manufacturer's catalogue is a conservative value, therefore the chances of failure of bearing is very less.

Equivalent radial load

The load rating of a bearing is given for radial loads only. Therefore, if a bearing is subjected to both axial and radial load, then an equivalent radial load is estimated as,

$$P_e = VP_r \quad \text{or}$$

$$P_e = XVP_r + YP_a$$

( )

Where,

P_e : Equivalent radial load
P_r : Given radial load
P_a : Given axial load
V : Rotation factor (1.0, inner race rotating; 1.2, outer race rotating)
X : A radial factor
Y : An axial factor

The values of X and Y are found from the chart whose typical format and few representative values are given below.

$\frac{P_a}{C_o}$	e	$\frac{P_a}{P_r} \le e$		$\frac{P_a}{P_r} \ge e$	
		X	Y	X	Y
0.021	0.21	1.0	0.0	0.56	2.15
0.110	0.30	1.0	0.0	0.56	1.45
0.560	0.44	1.0	0.0	0.56	1.00

The factor, C_o is obtained from the bearing catalogue.

The selection procedure

Depending on the shaft diameter and magnitude of radial and axial load a suitable type of bearing is to be chosen from the manufacturer's catalogue, either a ball bearing or a roller bearing. The equivalent radial load is to be determined from equation (14.2.3). If it is a tapered bearing then manufacturer's catalogue is to be consulted for the equation given for equivalent radial load. The value of
dynamic load rating C is calculated for the given bearing life and equivalent radial load. From the known value of C, a suitable bearing of size that conforms to the shaft is to be chosen. However, some augmentation in the shaft size may be required after a proper bearing is chosen.

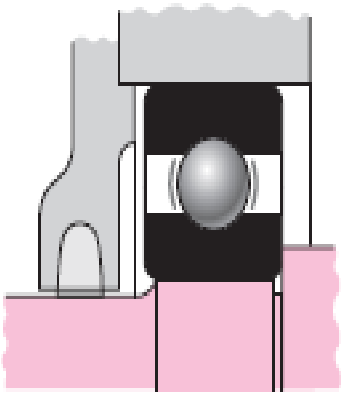
Seals

To exclude dirt and foreign matter and to retain the lubricant, the bearing mountings must include a seal. The three principal methods of sealings are the felt seal, the commercial seal, and the labyrinth seal (Fig.).

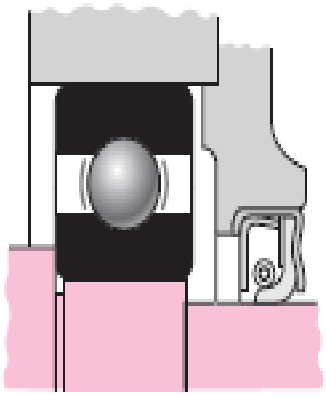
Felt seals may be used with grease lubrication when the speeds are low. The rubbing
surfaces should have a high polish. Felt seals should be protected from dirt by
placing them in machined grooves or by using metal stampings as shields.

The *commercial seal* is an assembly consisting of the rubbing element and, generally,
a spring backing, which are retained in a sheet-metal jacket. These seals are
usually made by press fitting them into a counterbored hole in the bearing cover. Since they obtain the sealing action by rubbing, they should not be used for high speeds.

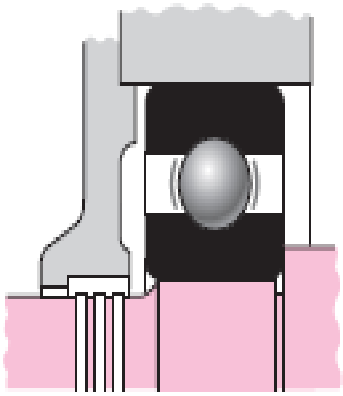
The *labyrinth seal* is especially effective for high-speed installations and may be used
with either oil or grease. It is sometimes used with flingers. At least three grooves should
be used, and they may be cut on either the bore or the outside diameter. The clearance
may vary from 0.010 to 0.040 in, depending upon the speed and temperature.



(*a*) Felt seal



(*b*) Commercial seal



(*c*) Labyrinth seal